Contents

QUANTITIES, THEIR SYMBOLS AND UNITS

Quantity	Symbol	Unit
Absolute pressure	p_{abs}	N/m^2 = Pa, bar
Absolute temperature	T	K
Work	W	Nm, J
Atmospheric pressure	p_{amb}	N/m^2 = Pa, bar
Acceleration	a	m/s^2
Work done for acceleration	W_a	Nm
Kinetic energy	W_{kin}	Nm
Bending stress	σ_b	N/mm^2
Shear strength	τ_{aB}	N/mm^2
Temperature in Celsius	ϑ	°C
Strain	ε	1
Density	ρ	kg/m^3
Work for turning	W	Nm
Turning power	P	W
Torque	M, M_d	Nm
Frequency of rotation	n	Min^1
Pressure	p	N/m^2 = Pa, bar
Pressure energy	W	Nm, J
Compressive stress	σ_d	N/mm^2
Diameter	d	m, mm
Electrical work	W	Ws, kWh
Electrical energy	W	Ws, kWh
Electrical charge	Q	As
Electrical power	P	W
Electrical potential	U	V
Electrical current	I	A
Electrical resistance	R	Ω
Energy	W, Q, E	Nm, J, Ws
Acceleration due to gravity	g	m/s^2
Surface pressure	σ_p	N/mm^2
Speed	v	m/s
Weight	F_G	N
Dynamic frictional force	F_R	N
Coefficient of dynamic friction	μ	1
Static frictional force	F_{R0}	N
Coefficient of static friction	μ_0	1
Hydraulic power	P	W
Hydrostatic pressure	p	N/m^2 = Pa, bar
Kinetic energy	W_{kin}	Nm
Buckling stress	σ_K	N/mm^2
Force	F	N
Diameter	d	m, mm
Circumference	l_u	m, mm
Length	l	m
Coefficient of linear expansion	α	m/(m • K) = 1/K
Power	P	W, kW
Mass	m	kg
Mechanical work	W	Nm, J

Mechanical energy	W	Nm, J
Mechanical power	P	W
Mechanical efficiency	η	1, %
Normal force	F_N	N
Standard acceleration due to gravity	g_n	m/s^2
Potential energy	W_{pot}	Nm
Cross section	S, A	mm^2
Radius	r	m, mm
Frictional force	F_{R0}, F_R	N
Coefficient of friction	$\mu_0,\ \mu$	1
Resultant force	F_r	N
Shear stress	τ_t	N/mm^2
Cutting speed	v_c	m/s, m/min
Safety factor	v	1
Specific resistance	ρ	$\Omega \bullet mm^2/m$
Yield point	R_e	N/mm^2
Flow velocity	w	m/s
Difference in temperature	$\Delta\vartheta, \Delta T$	°C, °K
Thermodynamic temperature	T	K
Torsional stress	τ_t	N/mm^2
Overpressure	p_e	N/m^2 = Pa, bar
Frequency of rotation	n	s^{-1}
Circumferential speed	v_u	m/s
Volume	V	m^3
Volumetric expansion coefficient	γ	$m^3/(m^3 \cdot K) = 1/K$
Flow rate	$V°$	m^3/s
Coefficient of thermal expansion	α	$m/(m \cdot K) = 1/K$
Heat = heat energy	Q	J, kJ
Distance	s	m
Angular velocity	ω	$rad/s = s^{-1}$
Efficiency	η	1, %
Time	t	s, min, h
Tensile strength	R_m	N/mm^2
Tensile stress	σ_z	N/mm^2

If one considers the tasks an **industrial foreman** has to complete in the different areas of a company, one can see that the principles behind many of the processes can be traced back to the **principles of physics**. Physics is an empirical science which has ancient roots but has developed particularly over the last 400 years. It is divided into different branches – as we can see in the following table.

Table 1: The development of physics

Branch	Development
The mechanics of solid bodies	since ancient times, the 16th century
Fluid mechanics	since ancient times, the 17th century
Optics	since ancient times, the 17th century
Acoustics	since ancient times, the 18th century
The mechanics of oscillations and waves	the 19th and 20th centuries
Thermodynamics	the 19th and 20th centuries
Electrical science	the 19th and 20th centuries
Atomic physics	the 20th century

In this booklet we consider the rules and laws of
The mechanics of solid bodies,
Fluid mechanics
Thermodynamics and
Electrical science

We will take particular care to orient the material on practical uses. Because of the abundance of material we could cover, it has been necessary to choose the most essential laws. Even though we vary from the normal presentation of physical problems, you can be assured that we will cover and practice everything necessary for your training.
We hope you enjoy working through this material, and that it brings you success!

1. LINEAR AND ROTARY MOTION

Solid objects can move in different ways. The aim of this unit is to consider the differences between these types of movement. You will certainly have encountered different types of motion at work, for example lifting a weight or the turning of a cogwheel, and will have noticed that one can differentiate then with time and special criteria.

Note: Time criteria are criteria on the state of movement.

Example:
Unchanging movement the speed is constant
Changing movement the speed is different at different times.

Note: The special criteria are those about the shape of the trajectory (line of movement).

Example:
Linear motion the direction stays constant.
Curvilinear motion the direction is constantly changing.
Special case: Movement in a circle.

1.1 Quantities and units

As you already know, relationships between scientific and technical quantities are almost always displayed in the shortest possible way, via **formulas**. The formulas are made up of **symbols.** The decisive norm for this is **DIN 1304 "symbols".** Due to the international connections in science, technology and trade, it was important to create a system of units that was internationally valid. This is the **SI system** which is built up from its **basic units**.

Table 1.1: Basic quantities and basic units

Quantity	Length	Mass	Time	Electric current	Thermodynamic temperature	Luminous intensity	Amount of substance
Basic unit	Metre	Kilogram	Second	Ampere	Kelvin	Candela	Mole
Symbol	m	kg	s	A	K	cd	mol

Note: All derived quantities are based on these seven basic units.

Example 1.1:
Definition formula relationship of units derived unit

speed = $\frac{distance}{time} \rightarrow$ v = $\frac{s}{t} \rightarrow [v] = \frac{[s]}{[t]} = \frac{metres}{second} = \frac{m}{s}$

It should also be noted that every physical quantity consists of a number and a unit, for example 5km, 3.2km or 6A.

1.2 The relationship between distance and time for linear motion

As you already know, the speed of a body can remain constant. One uses the phrase "uniform motion". If the speed changes over time, i.e. when there is acceleration or deceleration, then one talks about "non-uniform motion". As we are assuming linear motion here, one can say:

Note:	In uniform linear motion, a body moves with a constant speed in a straight line.

As you know from Mathematics, it is possible to represent the relationship between two quantities in a diagram. In kinetics (the study of motion), we do this in particular with **distance/time diagrams** and **velocity/time diagrams:**

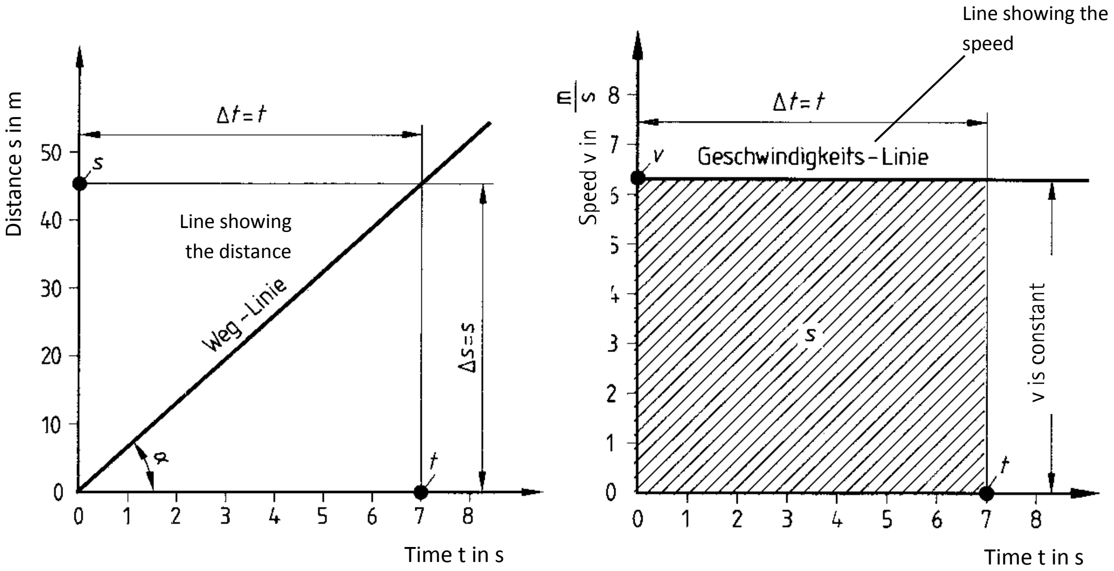


diagram of *s* and *t* for *v* = constant b) diagram of *v* and *t v* = constant
(distance, time diagram) (velocity, time diagram)
Figure 1.1: s, t diagram and v, t diagram for uniform linear motion
Figure 1.1 shows the relationships from example 1.1

Note:	The term *speed* is used to denote the quotient of the distance travelled by the object *s* = *s* and the time taken t = t.

Speed $v = \frac{s}{t}$ in $\frac{m}{s}, \frac{m}{min}, \frac{km}{h}$

Distance s = v • t in m, km

Time $t = \frac{s}{v}$ in s, min, h

One can see from the *v, t* diagram (figure 1.1b) that:

Note:	In the *v*, *t* diagram, the area under the line corresponds to the distance travelled by the body.

Example 1.2:

a) Describe the relationship between the units of time and distance.

b) A fork lift moves a distance of 100m uniformly in 10s. What is the value of its constant speed *v* in m/s and in km/h?

Solution:

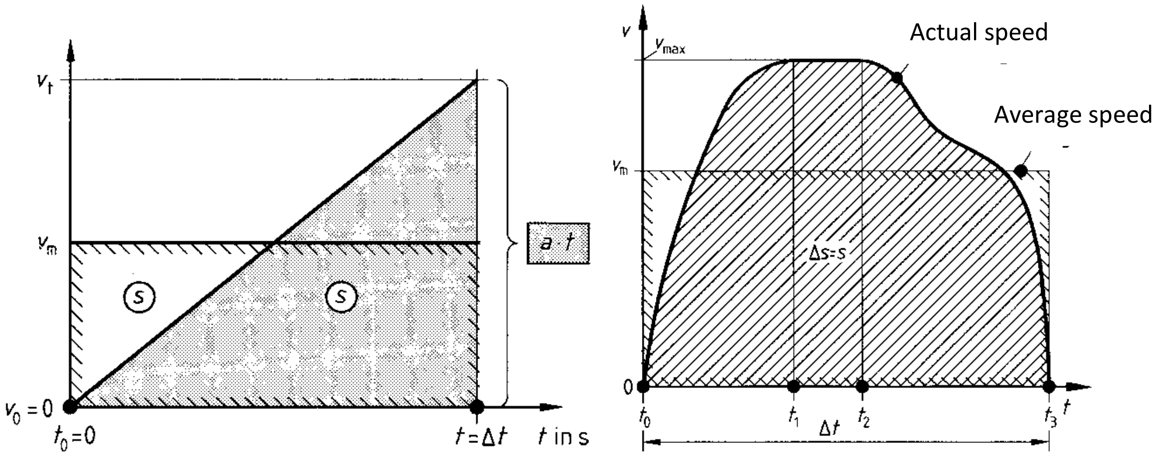
a) 1 min = 60 s   1 h = 60 min = 3,600 s

b) $v = \frac{s}{t} = \frac{100\ m}{10\ s} = 10\ \frac{m}{s}$

$10\ \frac{m}{s} = 10 \bullet \frac{3{,}600\ km}{1{,}000\ h} = 36\ \frac{km}{h}$ = 1 m/s = 3,6 km/h

Note:	When a body undergoes *non-uniform linear motion*, the speed of the body changes: the body accelerates or decelerates.

This can be constant or non-constant:



a) constant acceleration deceleration

b) non-constant acceleration and

Figure 1.2: v, t diagram of constant and non-constant acceleration

Note:	The term "acceleration" *a*, (or deceleration *a*) is used to denote the quotient of the difference in speed *v* and the time taken *t* = *t*.

Acceleration $a = \frac{\Delta v}{\Delta t} \rightarrow [a] = \left[\frac{\Delta v}{\Delta t}\right] = \frac{m/s}{s} = \frac{m/s}{s/1} = \frac{m}{s} \bullet \frac{1}{s} = \frac{m}{s^2}$

Note:	The derived unit for acceleration is meters per second squared.

Example 1.3:

A car accelerates from 0 to 100km/h in t = 9.1s.

a) What is the difference in speed in m/s?

b) Calculate the acceleration in m/s^2.

Solution:

a) $\Delta v = 100\,\frac{km}{h} = \frac{100\ m}{3.6\ s} = 27.7\overline{7}\ \frac{m}{s}$

b) $a = \frac{\Delta v}{\Delta t} = \frac{27.7\overline{7}\ \frac{m}{s}}{9.1\ s} = 3.0525\ \frac{m}{s^2}$

Generally, the following notation is used in kinetics:

Starting speed v_0

End speed v_t

The equation defining acceleration and Fig 1.2 for constant acceleration give the following derivation of the final speed v_t:

$a = \frac{\Delta v}{\Delta t} = \frac{v_t - v_0}{t - t_0}$ When accelerating from **rest**, v_0 = 0 and t_0 = 0. Thus:

$a = \frac{v_t - 0}{t - 0} = \frac{v_t}{t}$ this gives the acceleration from rest from the

end speed v_t = a • t in $\frac{m}{s}$, a in m/s^2 , t in s

Example 1.4:

A hoist accelerates to a = 1.3 m/s^2 in t = 1.9s.

How high is the end speed in m/min?

Solution:

$v_t = a \cdot t$ = 1.3 m/s^2 • 1.9 s = 2.47 m/s = 2.47 • 60 m/min = 148.2 m/min

Figure 1.2 shows us a simple geometric method for calculating the distance when the acceleration is constant and v_0 = 0 i.e. using the

area of a triangle → $s = \frac{v_t \cdot t}{2}$ or the

area of a rectangle → $s = v_m \cdot t = \frac{v_t}{2} \cdot t = \frac{v_t \cdot t}{2}$

If one puts $v_t = a \cdot t$ into these equations, one can see that the distance travelled is:

$s = \frac{v_t}{2} \cdot t = \frac{a \cdot t \cdot t}{2} = \frac{a}{2} \cdot t^2$

Thus:

the distance travelled when v_0 = 0: $s = \frac{v_t \cdot t}{2}$ or $s = \frac{a}{2} \cdot t^2$

So, we have dealt with an important case of non-uniform linear motion: that with $\boldsymbol{v_0}$ **= 0**. Additionally we have shown the value of drawing graphs of motions. This significantly improves the clarity and so one is able to derive the correct equations for the relevant motion easily. As a further example, consider figure 1.3, the *v,t* diagram of two motions with **constant acceleration** (or deceleration), namely with $v_0 > v_t$ with v_t not equal to 0 and $v_0 < v_t$ with v_0 not equal to 0:

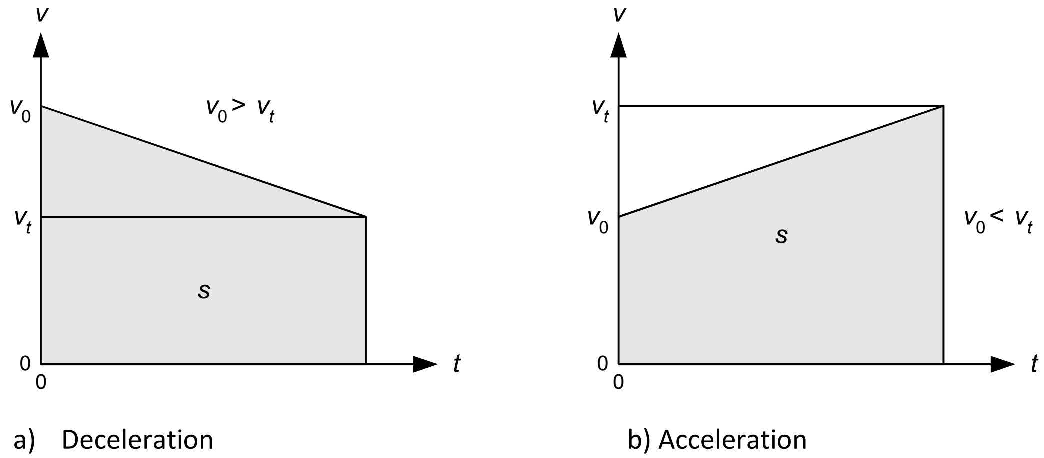


Figure 1.3: Motion with constant deceleration and constant acceleration

A free falling object, for example a stone falling from a tower, constantly accelerates towards the Earth. One uses the term **free-fall** and the acceleration which the body experiences is called the **acceleration due to gravity**. The **acceleration due to gravity** is a natural variable with the symbol *g*. It depends on where you are on the Earth, lying between 9.78 m/s^2 and 9.83 m/s^2. In our latitudes in Germany, we have the

acceleration due to gravity $g \approx 9.81$ m/s^2

We talk about a **normal acceleration due to gravity** of g_n **= 9.80665 m/s^2**, for rough calculations the value g = 10 m/s^2 is also used. Let us return to the example of the free falling stone. We note that the motion has a constant acceleration. Thus we can use the laws of constant acceleration. And we must merely note that:

Note:	The law of free fall works only in a vacuum i.e. when there is no air resistance.

This is clear because as the speed increases so does the air resistance which will then also influence the **speed of the fall.** For example, a free falling person (with unopened parachute) reaches a maximum speed of between 200 and 220 km/h.
In most technical situations, however, **air resistance** plays such a small role it can be ignored.
We use different **symbols for describing free fall** than for normal constant acceleration see table 1.2.

Table 1.2: Free fall treated as constant acceleration

General symbol	Symbol for free fall	Unit
Distance *s*	Distance fallen *h*	m
Time *t*	Time *t*	s
Speed *v*	Speed *v*	m/s
Acceleration *a*	Acceleration *a*	m/s^2

This gives us analogous formulas for free fall as for general constant acceleration.

Distance fallen $h = \frac{v_t \cdot t}{2} = \frac{v^2}{2g}$ in m

Final speed of the free fall $v_t = \sqrt{2 \cdot g \cdot h}$ in m/s

Condition: $v_0 = 0$

Example 1.5:
A workpiece falls 0.3m freely when being fitted to a machine.
a) For how long does it fall?
b) What is the final speed v_t?

Solution:

a) $h = = \frac{g}{2} \cdot t^2 \uparrow \sqrt{\frac{2 \cdot h}{g}} = \sqrt{\frac{2 \cdot 0.3\text{ m}}{9.81\text{ m/s}^2}} = 0.2473$ s

b) $v_t = \sqrt{2 \cdot g \cdot h} = \sqrt{2 \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot 0.3\, m} = 2.4261$ m/s

Check: $g = \frac{\Delta v}{\Delta t} = \frac{2.4261\, m/s}{0.2473\, s} = 9.81\text{ m/s}^2$

Note that **throwing directly upwards** is the opposite type of motion to free fall, we have to set $v_0 > 0$ and $v_t = 0$.

1.3 The laws of rotary motion

This unit looks at the relationship between translation and rotation. You will learn to transfer the laws of motion for linear motion over to rotary motion using analogies. The disk in figure 1.4 moves with such a rotary motion:

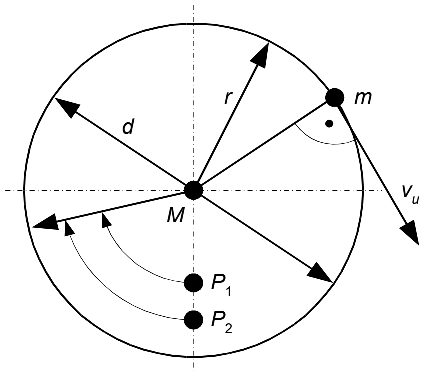


Figure 1.4: Rotary motion and circumferential speed

The trajectories of the individual points (e.g. P_1 and P_2) are **concentric circles,** i.e. circles with the same centre, the **rotation centre** M. The speed of the points depends on the distance from the rotation centre. This is clear because different points travel different distances in the same time. We can define the following based on figure 1.4.

Diameter	d = 2 • r	in m *r* = radius of the circle
Circumference	$l_u = \pi \bullet d$	in m = 3.1415... = **pi**

The speed of a point P is called the **rotary speed** it is greatest at the edge of the disk.

Note: The rotary speed at the circumference is called the circumferential *speed* v_u. It is always at right angles to the radius, i.e. *tangential.*

The **value of the rotary speed** does not only depend on the distance from the middle point but also on whether the **rotary system** is turning quickly or slowly. An important parameter is the **frequency of the rotation** for which we use the symbol *n*.

Note: The frequency of the rotation is the number of revolutions in the time Δt.

We can choose seconds or minutes for the period:

Frequency of the rotation

$n = \frac{i}{\Delta t}$

i = actual number of revolutions

n = number of revolutions per second or minute

The distance travelled in a revolution *s* is the circumference of the circle l_u, so $s = \pi \bullet d$.
As the frequency of the rotation depends on the time period Δt **= 1 s** and the distance for *n* rotations is $s = \pi \bullet d \bullet n$ one has that

$n = \frac{s}{\Delta t} = \frac{\pi \bullet d \bullet n}{1}$, this means that the

circumferential speed is $v = \pi \bullet d \bullet n$ in m/s ***d* in m, n in s^{-1}**

The time period generally used in mechanical and systems engineering is Δt = 1 min, and one talks about the frequency *n* in min^{-1}. The diameter is usually given in mm. In this case, the **circumferential speed** is $v = \pi \bullet d \bullet n$ ***d* in mm, *n* in $\frac{1}{min}$ = min^{-1}**

As 1m = 1000mm and 1min = 60s when using this time period Δt = 1 min the

circumferential speed is $v_u = \frac{\pi \bullet d \bullet n}{1{,}000}$ **in m/min**

circumferential speed is $v_u = \frac{\pi \bullet d \bullet n}{1{,}000 \bullet 60}$ **in m/s**

} ***d* in mm, *n* in min^{-1}**

In **production engineering** these formulas are differentiated by giving the **surface speed** v_c.

This is given, for example ...
... v_c for turning, planing, milling and drilling in m/min,
... v_c for polishing in m/s.

By proceeding appropriately, one can get useful numbers for the different use cases. These can easily be transformed as

$$1\frac{m}{s} = 60\frac{m}{min}$$

Example 1.6:
A grinding disc with diameter *d* = 180 mm has a frequency of rotation of *n* = 710 min^{-1}. What is its circumferential speed in m/s and in m/min?

Solution:

$$v_u = \frac{\pi \cdot d \cdot n}{1{,}000 \cdot 60} = \frac{180 \cdot \pi \cdot 710}{1{,}000 \cdot 60} = 6.69\frac{m}{s}, \qquad v_u = 6.69\frac{m}{s} \cdot 60\frac{s}{min} = 401.5\frac{m}{min}$$

Like linear motion we also differentiate here between
- constant rotary motion and
- non-constant rotary motion

If we are considering a machine and do not consider turning it on or off, we are normally talking about **uniform rotary motion.** Here we only consider such rotary motion, as displayed in figure 1.5:

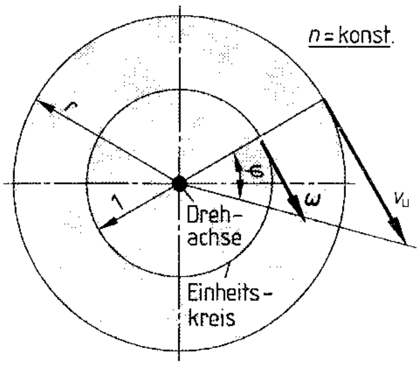


Figure 1.5: Uniform rotary motion

Note:	We use the term frequency to denote the number of revolutions, n, in the time period Δt.

Figure 1.5 shows the circumferential speed for **constant *n***, which depends on the radius. It is usual to give the **circumferential speed of the unit circle**, i.e. a circle with radius *r* = 1. This can be calculated from the **angle of rotation** through which the body turns in the period. We use the phrase **angular velocity.** The symbol for this is ω. In SI units:

Note:	1 *Radian per second* is the *angular velocity* of a uniformly rotating body which in 1s turns about its axis of rotation with an angle of 1 rad.

Remember:
Angles can be measured in degrees, minutes and seconds, but also in **radians**. If we take a sector of a circle and the length of the arc is the same as the radius of the circle, then the angle at the centre is **α = 57.29578.** We call this angle 1 r**adian** = 1 rad.

From this definition, the SI units give us that the

angular velocity $\omega = \frac{\Delta\varphi}{\Delta t}$ in $\frac{rad}{s} = s^{-1}$ **angle of rotation** $\varphi = \omega \bullet t$ in rad

For a full circle, i.e. a full revolution φ **=2 π rad** und $t = \frac{1}{n}$

thus $\omega = \frac{\Delta\varphi}{\Delta t} = \frac{\varphi}{t} = \frac{2\pi}{\frac{1}{n}}$ for the

angular velocity $\omega = 2 \bullet \pi \bullet \text{n}$ in in $\frac{rad}{s} = s^{-1}$

***n* = frequency of the rotation in s^{-1}**

If, as usual we put the **frequency of rotation *n* in min^{-1}**, then the previous equation give us $\omega = \frac{2 \bullet \pi \bullet \text{n}}{60}$, i.e. for the

angular velocity $\omega = \frac{\pi \bullet \text{n}}{30}$ in in $\frac{rad}{s} = s^{-1}$

***n* = frequency of rotation in min^{-1}**

Figure 1.5 shows us by geometric similarity that:

circumferential speed $v_u = \omega \bullet r$ in m/s

ω in s^{-1}, *r* in m

Note:	The circumferential speed v_u is the product of the angular velocity ω and the radius *r*.

Example 1.7:
A wheel with a diameter of *d* = 650 mm spins with *n* = 120 min^{-1}. Calculate
a) the angular velocity ω,
b) the circumferential speed v_u.

Solution:

a) $\omega = \frac{\pi \bullet \text{n}}{30} = \frac{\pi \bullet 120}{30} s^{-1} = 12.5664\ s^{-1}$

b) $v_u = \omega \bullet r = 12.5664 \bullet \frac{0.65}{2} = 4.0841 \frac{m}{s}$

Check: $v_u = \frac{d \bullet \pi \bullet \text{n}}{60 \bullet 1{,}000} = \frac{650 \bullet \pi \bullet 120\text{m}}{60 \bullet 1{,}000} \frac{m}{s} = 4.0841 \frac{m}{s}$

2. WORK, POWER, EFFICIENCY

In this section we consider the difference between work and power. We will also introduce a term which says something about the quality of a machine: efficiency.

2.1 Force, mass, friction and torque

Force is needed to accelerate a body. One can feel this oneself, for example, when in a car which is accelerating or decelerating. These forces can be very large and in the case of a car accident can only compensated for by a seat belt. This is formulated as **Newton's Second Law**:

Note:	If *F* is the force on an object of mass *m*, and *a* is the object's acceleration then *F* is equal to the product of *m* and *a*.

The law is named after the important English physicist Isaac Newton,

Force F = m • a $[F] = [m] \bullet [a]$ = kg • $\frac{kgm}{s^{-2}}$

Note:	The force which makes a mass of 1 kg accelerate by 1 m/s^2, is called 1 Newton = 1 N.

1 Newton = 1 kg • 1 m/s^{-2} = 1 $\frac{kgm}{s^{-2}}$

We have already seen that the **weight** of an object F_G, is a **gravitational force** directed to the middle of the earth and accelerates masses (free fall). As the **acceleration is due to gravity** g, there is an analogous equation for the force due to gravity:
Gravitational force F_G = m • g in N

Note:	The gravitational force is the product of the mass and the acceleration due to gravity.

Example 2.1:
A ball made from an alloy has the diameter *d* = 10 cm. The weight of the ball is F_G = 28 N. What is the ball's density ρ?

Solution:

$\rho = \frac{m}{V}$ (see Exercise 2), $V_{sphere} = \frac{\pi}{6} \bullet d^3$

$$\rho = \frac{\frac{F_G}{g}}{V} = \frac{F_G}{g \bullet V} = \frac{F_G}{g \bullet \frac{\pi}{6} \bullet d^3} = \frac{6 \bullet F_G}{g \bullet \pi \bullet d^3} = \frac{6 \bullet 28 \bullet kgm/s^{-2}}{9.81\, m/s^{-2} \bullet \pi \bullet (0.1m)^3} = 5{,}451.2\, \frac{kg}{m^3}$$

When two bodies touch, then **frictional forces** are created at the touching surfaces. These can be useful (e.g. in brakes) or unhelpful (e.g. when trying to push objects along the ground).

Note:	Friction on the external surfaces of a body is called *external friction*. The force is always in the opposite direction to any movement of the surfaces over each other.

We classify friction into **static friction** and **kinetic friction.** We explain with an example of a tailstock (figure 2.1):

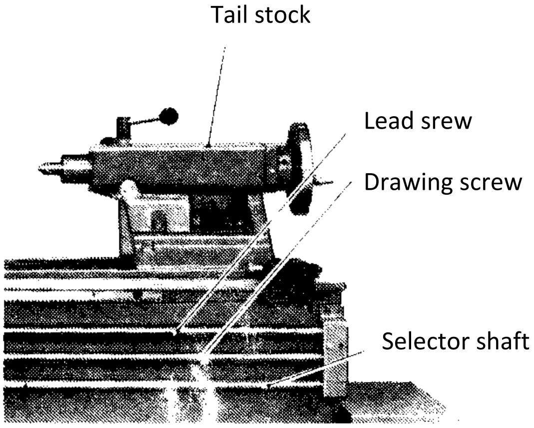


Figure 2.1: friction between solid objects

When working, a tail stock needs to be pushed along the bed of the lathe. One can feel that the force needed to start the tailstock moving is greater than the force needed to keep it sliding along the bed. Thus we differentiate the friction forces like so:

Static friction F_{R_0} friction when there is no movement
Kinetic friction F_R friction when there is movement

As mentioned, friction is always in the opposite direction to movement. It is created by movement, so it is a **reaction.** Figure 2.2 shows the forces on the tail stock systematically:

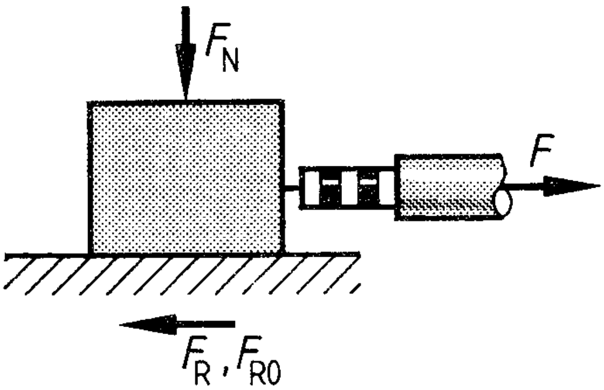


Figure 2.2: Friction and normal force

Note: *Action F* (measurable) is equal to the *reaction* F_R (kinetic friction) or F_{R_0} (static friction)

If one does the experiment depicted in figure 2.2, one can see that the friction (F_R = *F* or. F_{R_0} = *F*) is proportional to the force acting at right angles between the two surfaces. This is called the normal force F_N.

Note: Friction is proportional to the normal force of a body on its base.

Of course, other factors influence friction too. In particular there is the material of which the surfaces are made (e.g. steel on cast iron) and the condition of the surfaces. These factors are brought together in the coefficient of the proportionality as the **coefficient of friction**. One differentiates between the

Static coefficient of friction μ_0 and Kinetic coefficient of friction μ

These coefficients of friction can only be found empirically. Table 2.1 shows some **averages.**

Table 2.1: Coefficients of friction

Material	Condition	Static coefficient of friction μ_0	Kinetic coefficient of friction μ
Cast iron/cast iron	lubricated	0.16	0.12
Steel/steel	dry	0.15	0.1
Steel/cast iron	lubricated	0.1	0.05
Steel/leather	dry	0.6	0.3
Wood/metal	lubricated	0.1	0.06
Steel/ice	dry	0.027	0.014

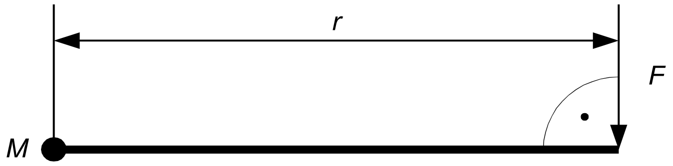
We can thus write the forces in terms of the coefficients μ_0 and μ:

Static friction $F_{R_0} = \mu_0 \bullet F_N$ in N
kinetic friction $F_R = \mu \bullet F_N$ in N

Example 2.2:
The sliding carriage of a shaping machine has a weight of F_G = 3,728 N. It moves on the upper surface of the machines bed. Both the carriage and bed are made of cast iron and are well lubricated. Thus μ_0 = 0.16 and μ = 0.1. Calculate
a) the static friction,
b) the kinetic friction.

Solution:
a) $F_{R_0} = \mu_0 \bullet F_N$= 0.16 • 3,728 N = 596.48 N
b) $F_R = \mu \bullet F_N$= 0.1 • 3,728 N = 372.8 N

We can assume that you have already worked with a spanner. This shows you that rotational effects do not only depend on the size of the force, but also on the length of the **lever** available. Figure 2.3 shows the situation schematically:



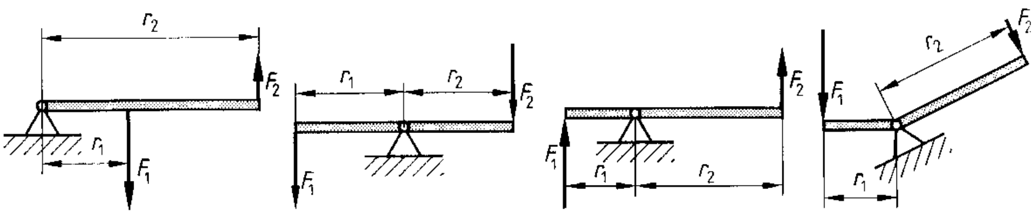
Force	F
Length of lever	r
Fulcrum	M

Figure 2.3: Lever

Note:	The moment, or torque is the product of the force *F* and the perpendicular distance *r* to the fulcrum *M*.

Torque $M = F \cdot r$ $[M] = [F] \cdot [r] = N \cdot m =$ **Nm**

Note:	The derived SI unit of torque is the Newton-meter Nm.



a) b) c) d)

Figure 2.4: Types of lever

Figure 2.4 shows different **types of lever.** From left to right: a one sided lever, a two sided lever with equal sides, a two sided lever with unequal sides and the angle lever.

Note:	The arms of a lever are measured from the point at which the force is applied to the fulcrum (the centre of rotation).

We can see from figure 2.4 that there are forces that cause **clockwise** rotation or **anticlockwise** rotation. So one talks about

- **clockwise torque** and
- **anticlockwise torque.**

Levers are used in tongues, scissors, wrenches, hinges, crowbars etc. We can see that in most cases there is an equilibrium **of moments**. We assume an equilibrium to formulate the law of the lever:

Note:	There is an equilibrium on a lever if the sum of the clockwise moments is equal to the sum of the anticlockwise moments.

To be able to use this in calculations we use the

sign rule:	**negative torque**	(–)	→ clockwise
	positives torque	(+)	→ anticlockwise

Law of the lever $\Sigma M = F_1 \cdot r_1 + F_2 \cdot r_2 + ... = 0$

Example 2.3:

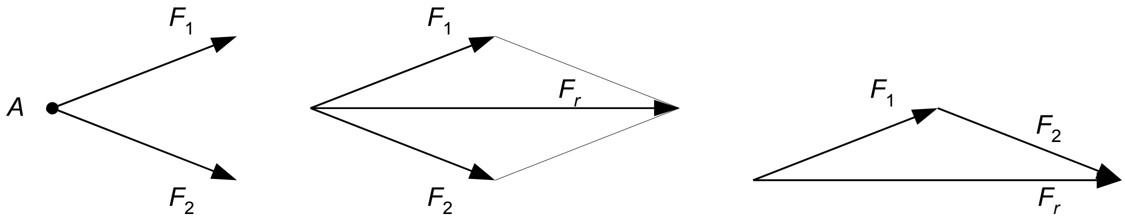
In part a) of Figure 2.4, F_1 = 85 N, r_1 = 65 cm, r_2 = 152 cm.
Calculate the force F_2 if there is an equilibrium.

Solution:

$$- F_1 \bullet r_1 + F_2 \bullet r_2 = 0 \rightarrow F_2 = F_1 \bullet \frac{r_1}{r_2} = 85\ \text{N} \bullet \frac{65\ cm}{152\ cm} = 36.35\ \text{N}$$

Note:	When dealing with diagonal forces (see Example 2.4) the component of the force perpendicular to the arm of the lever is to be used.

To be able to do this, we talk briefly about two forces which both act at the same point, and consider them as one force. This is done, as in figure 2.5, with the help of a **force parallelogram** or **force triangle.** The forces F_1 and F_2 put together as the **resulting force** F_r:



a) b) Force parallelogram c) Force triangle

Figure 2.5: Adding two forces which act at the same point.

Note:	The resulting force F_r acts at the same point as the individual forces, replacing them.

On the other hand, it is possible to split a force into two component forces. For example a **horizontal component** F_x and a **vertical component** F_y:

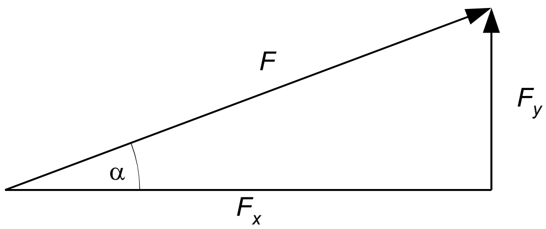


Figure 2.6: Breaking down a force into components

We can apply the laws of trigonometry to figure 2.6:

$\sin\alpha = \frac{F_y}{F}$	Vertical component	F_y = F • $\sin\alpha$
$\cos\alpha = \frac{F_x}{F}$	Horizontal component	F_x = F • $\cos\alpha$

Example 2.4:

In figure 2.7, F_1 = 500 N, F_2 = 150 N, α = 30°, r_1 = 50 cm, r_2 = 30 cm, r_3 = 20 cm. What must the force F_3 be to keep the lever in equilibrium?

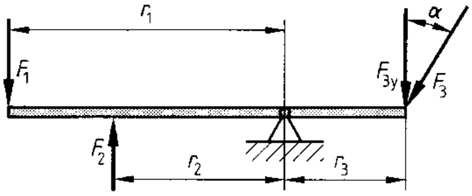


Figure 2.7: Vertical component F_{3y}

Solution:

$\Sigma M = 0 = F_1 \bullet r_1 - F_2 \bullet r_2 - F_{3y} \bullet r_3$ $\qquad$ $F_{3y} = F_3 \bullet \cos\alpha$ $\qquad$ Thus

$F_1 \bullet r_1 - F_2 \bullet r_2 \bullet \cos\alpha \bullet r_3 = 0$ $\qquad$ $\cos\alpha = \cos 30° = 0.86$

$$F_3 = \frac{F_1 \bullet r_1 - F_2 \bullet r_2}{r_3 \bullet \cos\alpha} = \frac{500\ N \bullet 50\ cm\ - 150\ N \bullet 30\ cm}{20\ cm \bullet 0.866} = 1{,}183.6 \text{ N}$$

2.2 Mechanical work and mechanical energy

In this unit, we will explain the relationships between work, energy and power. Further, we will consider the transformation of energy and the "quality" of machines and technical equipment.

In Physics, **mechanical work** is only performed when a force acts along a distance travelled. This means, when a body is pushed by a force. Such a situation can be seen in figure 2.8:

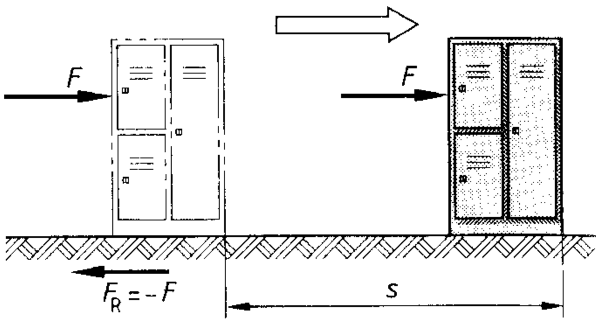


Figure 2.8: Mechanical work

Figure 2.8 shows friction again as a reaction to a **displacement force.**

Note:	Mechanical work is the product of the force *F* and the distance travelled in the direction of the force *s*. The symbol used is *W*.

Mechanical work $\qquad$ W = F • s $\qquad$ [*W*] = [*F*] • [*s*] = N • m = **Nm**

You will immediately notice that the unit of mechanical work is the same as for the torque, i.e. the Newton meter. However:

Note:	Mechanical work ($F\,s$) and torque ($F \cdot r$) are different quantities.

Note:	The derived *SI Unit for mechanical work* is the *Joule* (symbol: J). 1 J is the work done when a body is moved s = 1 m with a force of F = 1 N.

Mechanical work can be shown via a **force/distance graph** (*F,s-graph*). Figure 2.9 is such a graph showing a changing and occasionally constant force over a specific distance:

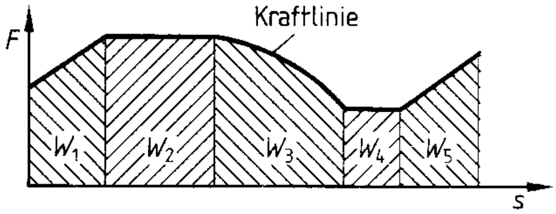


Figure 2.9: force/distance graph (F,s-graph)

Note:	In an F,s-graph, the area under the graph is the mechanical work performed.

Total work $F_{ges} = W_1 + W_2 + W_3 + \ldots$ in Nm, J (see figure 2.9)

Just like with the torque, we can only calculate the mechanical work with the **effective component** of the force. In figure 2.10 this is F_x:

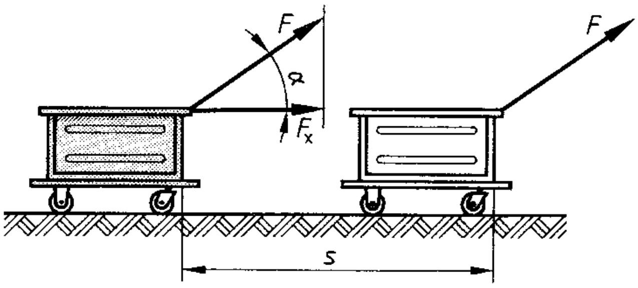


Figure 2.10: Effective component

Note:	When calculating mechanical work, the component of the force acting parallel to the distance travelled is to be used.

Example 2.5:

In figure 2.10, F = 100 N, α = 35° and s = 850 m. How much mechanical work is done?

Solution:

$W = F_x \cdot s = F \cdot \cos\alpha \cdot s$ = 100 N • $\cos 35°$ • 850 m = 100 N • 0.8192 • 850 m = 69,632 N

When mechanical work is done, e.g. lifting a weight (see figure 2.11), then after the work is done, the system's internal **energy** is raised by the amount of the work done. Here, the meaning of the term energy becomes clear as if the weight is let loose, it is able to fall down and carry out mechanical work in the process, e.g. to drive a stake into the ground.

Note:	The term "energy" is used to describe the potential to do work which is stored in a system.

There are different **types of energy**. For example: electrical, atomic, chemical, thermal, mechanical energy and so on. All types of energy are measured in the same units because any type of energy can be transformed into any other type. The amount of energy never changes. Thus, for example, **thermal energy** can be transformed into **mechanical energy** (in a turbine) and then into **electrical energy** (in a generator). In order to be able to distinguish between the types of energy by looking at the units, the following choice of units is often used:

Table 2.2: Energy

Type of energy	Unit
mechanical energy	Newton meter = Joule Nm, J
thermal energy	Joule J
electrical energy	Watt second, Kilowatt hour Ws, kWh

However, some of these units are equivalent:

equivalence of energy units 1 J = 1 Nm = 1 Ws

Figure 2.11 shows the example of lifting a weight.

Work done $W_h = F \bullet h$ in Nm h = height in m

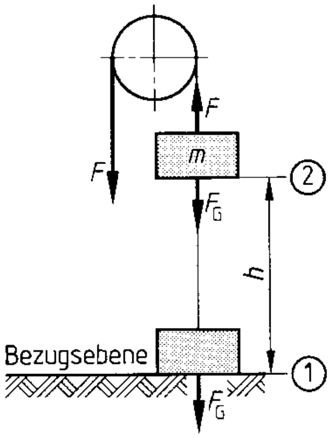


Figure 2.11: Lifting a weight

As we have already said, the energy of the mass *m* is greater when it has been lifted due to its higher position. The amount it has increased is the same as the amount of work done. One calls this energy

potential energy. Here we have $F = F_G = m \cdot g$ so the

potential energy $W_{pot} = F_G \cdot h = m \cdot g \cdot h$ in Nm F_G = force of gravity in N

Note:	The energy of a body related to its height is called its potential energy.

Recall the equation ***F* = *m* • *a*** which relates inertial force to the acceleration it produces. Figure 2.12 shows the action of such a force resulting in the acceleration of the trolley from a speed of v_0 to v_t:

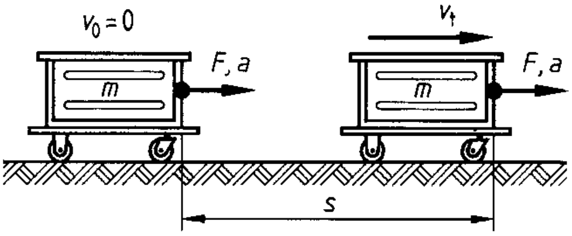


Figure 2.12: Force and acceleration

Recall that mechanical work is calculated with the equation ***W* = *F* • *s***. If the acceleration is constant at $a = \frac{v_2}{2 \cdot s}$, then we have

Work done during the acceleration $W_a = F \cdot s = m \cdot a \cdot s = m \cdot \frac{v_2}{2 \cdot s} \cdot s = \frac{m}{2} \cdot v^2$ in Nm

At the end of the motion depicted in figure 2.12, the potential to do work of the trolley and thus its energy have increased. The amount it has increased is the same as the work done to accelerate it. This energy is released, for example, in a collision. It is called **kinetic energy**.

kinetic energy $W_{kin} = \frac{m}{2} \cdot v^2$ $\rightarrow [W_{kin}] = \frac{[m]}{2} \cdot [v^2] = kg \cdot \frac{m^2}{s^2} = \frac{kgm}{s^2} \cdot m =$ **Nm**

Note:	The kinetic energy of a moving body is equal to one half times its mass times the square of its speed.

Example 2.6:

The mass depicted in figure 2.11 has *m* = 15kg, a height of *h* = 3.5m. Calculate

a) the potential energy W_{pot},

b) the final speed after a free fall *v*,

c) the kinetic energy W_{kin} just before it lands.

Solution:

a) $W_{pot} = F_G \cdot h = m \cdot g \cdot h = 15\ kg \cdot 9.81 \frac{m}{s^2} \cdot 3.5\ m = 515.025$ Nm

b) $v = \sqrt{2 \cdot g \cdot h} = \sqrt{2 \cdot 9.81 \frac{m}{s^2} \cdot 3.5\ m} = \sqrt{68.67 \frac{m^2}{s^2}} = 8.2867 \frac{m}{s}$

c) $W_{kin} = \frac{m}{2} \cdot v^2 = \frac{15\ kg}{2} \cdot (8.28674\ \frac{m}{s})^2 = 515.025$ Nm

Example 2.6 shows that energy can be transformed from one form into another. Here is it potential energy into kinetic energy. If we do not take account of any **energy loss** – e. g. due to air resistance during the free falls, then the energy transformed is the total and we use the phrase **conservation of energy.** In general, the **law of conservation of energy** states:

Note:	The energy at the end of a technical process is the energy at the start plus the energy added minus the energy taken out during the process.

2.3 Mechanical power and efficiency

You will know from your work that it is not just the amount of work that is done, but how quickly it is done that is important. It matters whether a process takes 1.3 hours or 1.7 hours. This leads us to the definition of **power**. Power is large when mechanical work is done in a short time:

mechanical power $P = \frac{W}{t} \quad \rightarrow [P] = \frac{[W]}{[t]} = \frac{Nm}{s} = \frac{Ws}{s}$ = W = **Watt**

Note:	Mechanical power is the quotient of mechanical work and time taken. The derived unit is the *Watt.*

1,000 Watts are called a **kilowatt** (kW). This is related to horsepower (HP):

1,000 W = 1 kW **1 kW = 1.36 PS**

If we substitute $W = F \bullet s$ into the power equation, we get

$P = \frac{F \bullet s}{t} = F \bullet \frac{s}{t}$ Using $v = \frac{s}{t}$ we see that

mechanical power $P = F \bullet v \quad \rightarrow [P] = [F] \bullet [v] = N \bullet \frac{m}{s} = \frac{Nm}{s} = \bullet \frac{Ws}{s}$ = **W**

Note:	Mechanical power *P* is the product of the force *F* and the speed generated by it *v*.

The equations for mechanical work, energy and power are not only applicable to linear motion but also to rotation. Thus we have

Power $P = F_u \bullet v_u$ in W F_u = force acting at the circumference in N
v_u = circumferential speed in m/s

Example 2.7:

A component is worked on a lathe. The diameter of the component is *d* = 1,500mm. The frequency of rotation is n = 90 min^{-1} and the force at the tool is F_c = 1,060N. Calculate

a) the circumferential speed in m/s,
b) the power P_c,
c) the work done in 5 minutes in both Nm and kWh (Kilowatt hours).

Solution:

a) $v_u = \frac{d \bullet \pi \bullet n}{1{,}000} = \frac{1{,}500 \bullet \pi \bullet 90}{1{,}000} \frac{m}{min}$ = 424.12 $\frac{m}{min}$ = $\frac{424.12}{60} \frac{m}{s}$ = 7.069 $\frac{m}{s}$

b) $P_c = F_u \bullet v_u$ = = $F_c \bullet v_c$ = 1.060 N • 7.069 $\frac{m}{s}$ = 7,493.14 $\frac{Nm}{s}$ = 7,493,14 W

c) $P_c = \frac{W}{t}$ → W = $P_c \bullet t$ = 7,493.14 $\frac{Nm}{s}$ • 5 • 60 s = 2,247,942 Nm

W = 2.247.942 Ws = $\frac{2.247.942}{1{,}000 \bullet 3{,}600}$ kWh = 0.6244 kWh

To overcome friction, work needs to be done which we denote by W_R. In Example 2.7, this is comprised of various frictional effects within the lathe which occur between the machine's motor and the tool, for example in all the bearings. The work supplied by the machine's motor is called the **work supplied** W_a where as the work applied by the tool is called the **effective work** W_n. These are linked by the following general rule:

energy and work balance $W_a = W_n + W_R$ So that we always have: $W_a > W_n$

We can use W_n and W_a to say something about the quality of a machine or piece of equipment. This relationship is called

mechanical efficiency $\eta = \frac{W_n}{W_a}$ → $\eta < 1$ as $W_a > W_n$

If we divide both W_n and W_a by the time t, which will be the same for both quantities, then we get the **effective power** P_n and the **power supplied** P_a. Thus we can also write

mechanical efficiency $\eta = \frac{W_n / t}{W_a / t} = \frac{P_n}{P_a} < 1$

Note: Mechanical efficiency can be calculated from the relationship between the supplied and effective work or the supplied and effective power.

Example 2.8:
The lathe in Example 2.7 has a motor supplying P_a = 10.5 kW.
Calculate the mechanical efficiency.

Solution:
$\eta = \frac{P_n}{P_a} = \frac{P_c}{P_a} = \frac{7.5\ kW}{10.5\ kW}$ = 0.714 = 71.4 %

The **efficiency** is often given as a **percent**. Then is says the percentage of energy or power used which is transformed into effective energy or power. The rest of the energy or power is not used usefully by the machine or equipment.

3. POWER TRANSMISSION

In this unit we consider the possibilities of **power transmission**. Firstly we must consider what is meant by the term power transmission. We will start with an example we have already seen: the lathe in Exercises 2.7 and 2.8. The power is transmitted here by coupling the electric motor directly to the drive. We call this **mechanical power transmission.** Generalizing this example we see:

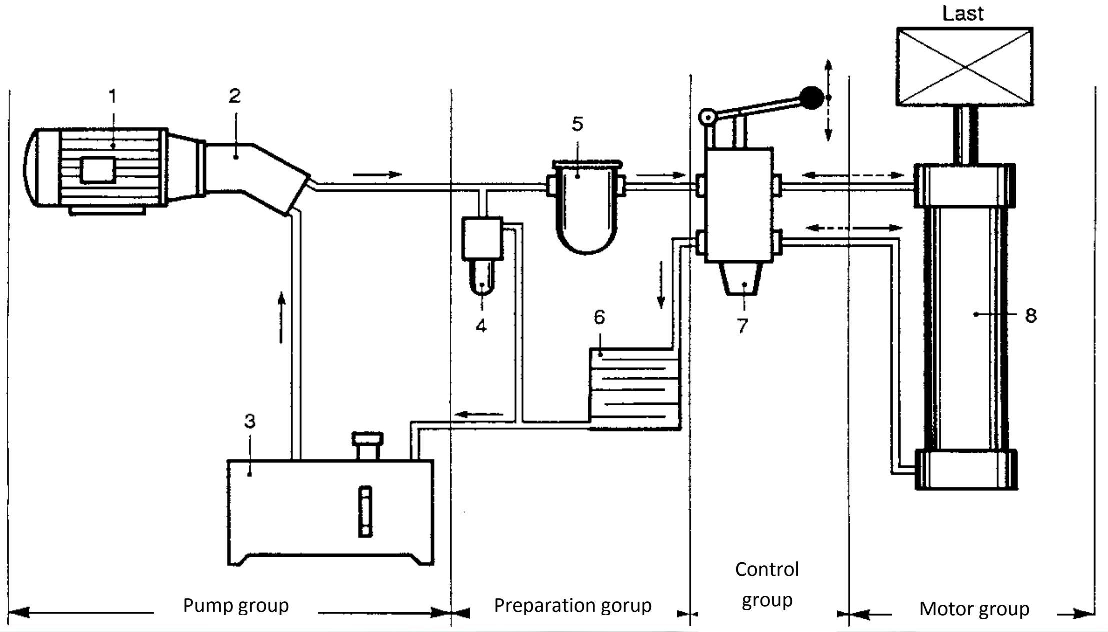
Note:	Power is always transmitted from a specific machine or piece of equipment to another, very often it is from an *engine* to another machine.

At one time, mechanical power transmission was dominant but now one often uses electrical, hydraulic or pneumatic power transmission. This often has great advantages. For example, hydraulic power transmission is significantly more sensitive than mechanical power transmission. This is an advantage in, for example, machine tool building.

Table 3.1: Different types of power transmission

	Mechanical	**Electrical**	**Hydraulic**	**Pneumatic**
Transmission particles	Atoms within a metal	Electrons within a metal	Molecules in a liquid	Molecules in a gas
Transmission devices	Chains, cogs, gears, shafts, belts etc.	Copper or aluminium wire	Copper or steel piping	Piping, tubes, hoses
Physical quantities	Force, distance, speed	Voltage, current, time	Pressure, volume, flow rate	Pressure, volume, flow rate

Power transmission can also involve combining types for example electrical and hydraulic. We can then talk about the individual **transmission components.** Figure 3.1 shows an example of hydraulic power transmission:



1 Electric motor 3 Container for oil 5 Filter 7 Directional valve
2 Water pump 4 Pressure relief valve 6 Oil cooler 8 Hydraulic cylinder

Figure 3.1: design of a hydraulic drive

The electric motor (1) drives a pump (2). The pump (2) sucks oil out of the container (3) and transfers it under pressure into the system. The pressure relief valve (4), also called a safety valve, keeps the pressure in the system under the maximal allowed level. The filter (5) cleans the oil in the system. By using the directional valves (7), the hydraulic cylinder (8) can be extended or retracted depending on the direction. Finally, the oil flowing from the control valve (7) to the container (3) is cooled in the oil cooler (6).

The transmission devices are split into **subassemblies** which we can see in figure 3.1:

- The **pump group** is the energy source of the hydraulic system. It contains the motor, the pump, the container for the oil and possibly a hydraulic accumulator.

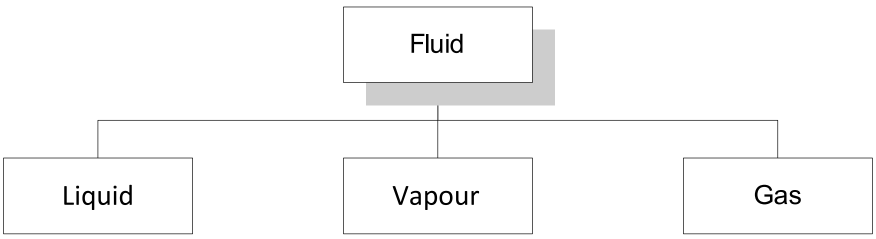
- The control group is for controlling and regulating the system, making sure that the **hydraulic medium** gets to the right place with the right volume, and pressure. It contains the directional valves and other components like, for example, flow and pressure control valves.

- The **preparation group** keeps the hydraulic medium and the system in an optimal condition. It contains filters, coolers and heaters. Flow and pressure control valves can also be classed in this group if desired rather than the control group.

- The **motor group** changes the hydraulic energy into mechanical energy and drives the load. It includes: hydraulic motor, cylinders and swivel motors.

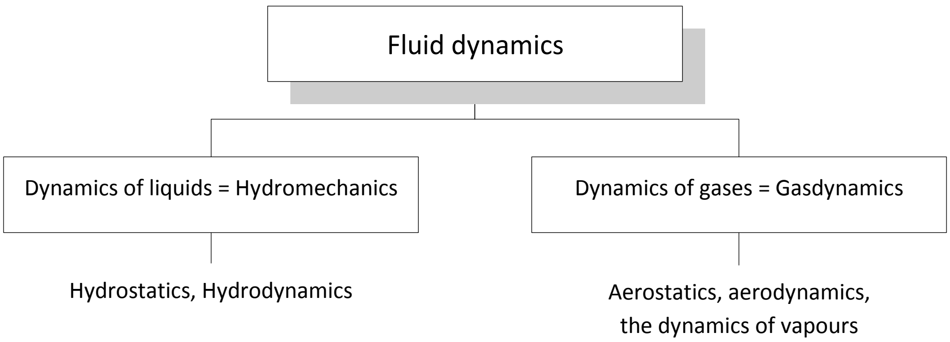
3.1 Fluids as working media for hydraulic and pneumatic systems

Bodies can be solid, liquid or gaseous. We refer to these forms as the **states of matter**. Liquids and gases (including vapour) are considered together as fluids. Fluids are often mixtures, for example humid air is a mixture of fluids. We divide them up in the following way:



Note:	Fluids do not have a fixed shape. Relatively small forces can change their shape.

Fluid dynamics is the study of how fluids behave when forces are applied to them. This is structured in the following way:



Fluids obey the **Navier–Stokes equations**. Unlike solids, fluids have no shape and take on the shape of their containers. Gases and vapours generally spread out to the whole space available. While liquids are generally **incompressible**, gases and vapours are **compressible.** These properties open up many fields of application, in particular hydraulic and pneumatic systems.

Note: The term *hydraulic* is used for technical processes and systems where *force or power transmission and/or control* is carried out by using liquids in closed systems. *Pneumatic* systems, on the other hand, use a gas or vapour, in many cases pressurized air.

A system in which hydraulic and pneumatic systems are coupled together for power transmission and control is called **pneudraulics.**

The term **control group** has already been mentioned. It is responsible for **open-loop and closed-loop control.** Now we need to explain the terms "open-loop" and "closed-loop".

DIN 19226 defines **open-loop control** in the following way:

Note: Open-loop control is the process where one or more input variables influence output variables due to the properties of the system. A characteristic of control is the *open sequence of actions* via the individual transmission devices or chain of control.
Control can be automatic or manual.

Closed-loop control is also defined in DIN 19226:

Note: Closed-loop control is a process which determines the *variables to be regulated*, compares them with *reference variables* and depending on the results adjusts the variables to be regulated according to the reference variables. The actions here take place in a *closed loop*. The regulation can be automatic or manual.

Just like open-loop control, closed-loop control influences the actuating variable and the actuator via the regulating equipment using a flow of matter or energy, see figure 3.2. However, the actuating variable is an output variable of the regulation equipment and so depends on the comparison of the variable to be regulated and the reference variable. Here we can see the circular structure of closed-loop control.

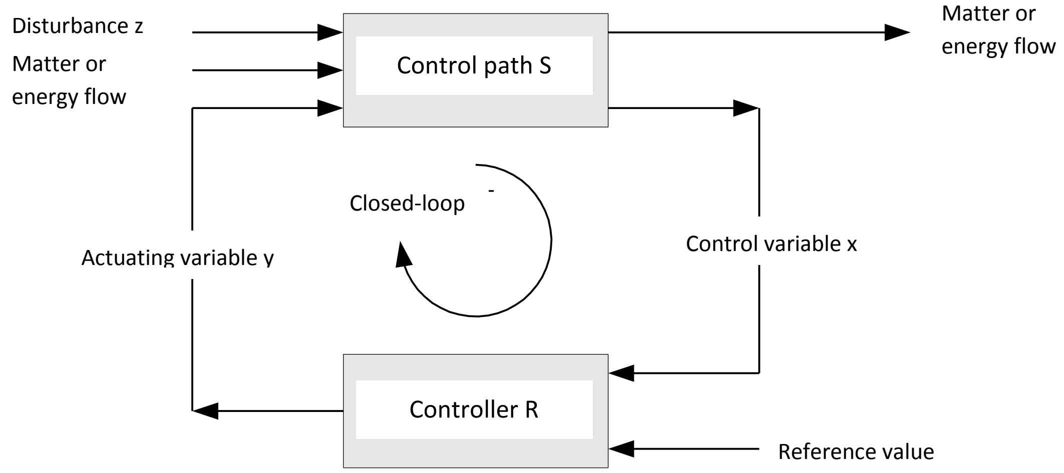


Figure 3.2: Circular structure of regulation

3.2 The important principles of hydraulics

It is important to distinguish between fluids that are not moving (**fluid statics**) and those that are (**fluid dynamics**).

Table 3.2: Subdivisions of fluid dynamics

Fluid	**Static**	**Moving**
Liquid	Hydrostatics	Hydrodynamics
Gas or vapour	Aerostatics	Aerodynamics

A good example of hydrostatics is when a liquid is in an enclosed space and is pressed by a piston. This is shown in figure 3.3. Note that the reaction *F′* equals the force from the piston *F*.

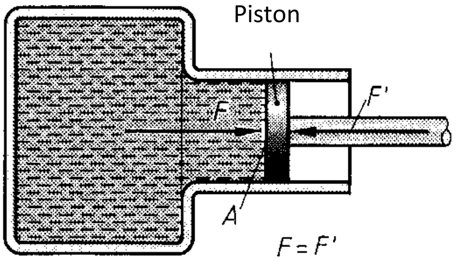


Figure 3.3: A piston pressing a liquid.

Note:	The quotient of the force *F* and the area pressed A is called the hydrostatic pressure and has symbol *p*.

hydrostatic pressure $p = \frac{F}{A}$ $\rightarrow [p] = \frac{[F]}{[A]} = \frac{\boldsymbol{N}}{\boldsymbol{m^2}}$

Note:	Hydrostatic pressure is measured in Newtons per square metre.

1 N/m^2 is also called 1 **Pascal** (Pa). This is a very small pressure: normal air pressure is about 10^5 Pa. For this reason, we also give the value 10^5 Pa its own unit: the bar. Thus

SI units of pressure 1 bar = $10^5 \frac{N}{m^2}$ = 10^5 Pa

We can see from the equation for hydrostatic pressure that

The force on a surface of area A is F = p • A $\rightarrow [F] = [p] \bullet [A] = \frac{\boldsymbol{N}}{\boldsymbol{m^2}} \bullet m^2 = \mathbf{N}$

Example 3.1:

There is a closed container (figure 3.3) with a pressure of *p* = 12 bar. What force does it have on a piston of diameter *d* = 60 mm?

Solution:

$p = \frac{F}{A}$ $\rightarrow$ F = p • A = $p \bullet \frac{\pi}{4} \bullet d^2 = 12 \bullet 10^5 \frac{N}{m^2} \bullet \frac{\pi}{4} \bullet (0.06m)^2$ = 3,392.92 N

We can see that the value of the force is directly proportional to the area of the piston. This principle of generating force is extremely useful, for example for pistons in combustion engines. A further important use is in **hydraulic power transmission,** for example hydraulic brakes or hoists. Figure 3.4 shows us a hydraulic cylinder again. The diagram is simplified, showing only the important elements: the pistons with their differing diameters *D* and *d*:

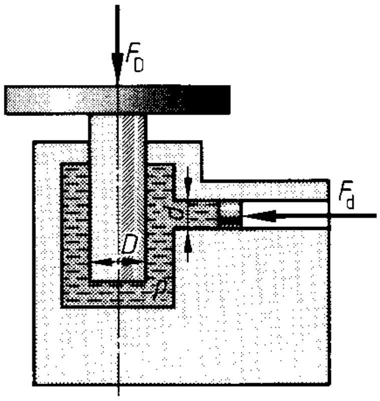


Figure 3.4: The principle behind hydraulic hoists

The pressure in the cylinder *p* affects both pistons so that:

$p = \frac{F_D}{A_D} = \frac{F_d}{A_d} = \frac{F_D}{\frac{\pi}{4} \bullet D^2} = \frac{F_d}{\frac{\pi}{4} \bullet d^2} = \frac{F_D}{D^2} = \frac{F_d}{d^2}$ Thus, the

force generated on the piston (assuming no losses) $F_D = F_d \bullet \frac{D^2}{d^2}$ $\rightarrow \frac{F_D}{F_d} = \frac{D^2}{d^2}$

Note: In *hydraulic power transmission,* the *forces* on a piston depend on the *square of the diameter of the piston.*

As we can assume that the media in hydraulic systems are almost incompressible, when one piston is displaced, another piston must also move so that the volume remains the same. In figure 3.4 – the distance travelled by the smaller piston is denoted s_d and that of the larger piston s_D. This gives us the following equations linking the volumes:

$$\left.\begin{aligned} V_d &= \frac{\pi}{4} \bullet d^2 \bullet s_d \\ V_D &= \frac{\pi}{4} \bullet D^2 \bullet s_D \end{aligned}\right\} \rightarrow \frac{\pi}{4} \bullet d^2 \bullet s_d = \frac{\pi}{4} \bullet D^2 \qquad \rightarrow \frac{D^2}{d^2} = \frac{s_d}{s_D}$$

Note:	In hydraulic power transmission, the relationship between the distances travelled by the pistons is the inverse to the relationship between the square of their diameters.

distance travelled by the piston $s_D = s_d \bullet \frac{d^2}{D^2}$

As the distances s_D and s_d are travelled by the pistons in the same time, we can use the law for constant linear motion $s = v \bullet t$

$v_D \bullet t = V_d \bullet t \bullet \frac{d^2}{D^2}$ and so

the piston speed generated $v_D = v_d \bullet \frac{d^2}{D^2} \qquad \rightarrow \frac{V_D}{V_d} = \frac{d^2}{D^2}$

Note:	In hydraulic power transmission the piston speeds behave in the inverse manner to the square of the piston diameters.

Example 3.2:

Two pistons in a hydraulic control system have diameters of *d* = 3 mm and *D* = 20 mm. There is a pressure of *p* = 1.2 bar in the connecting pipe. The control piston with diameter *d* is pushed at a speed of *v* = 1.5 m/s with a force of F_d = 10 N. Calculate the force on the other piston F_D and the speed it moves at v_D.

Solution:

$$F_D = F_d \bullet \frac{D^2}{d^2} = 10\text{ N} \bullet \frac{(20\ mm)^2}{(3\ mm)^2} = 10\text{ N} \bullet \frac{400\ mm^2}{9\ mm^2} = 444.44\text{ N}$$

$$v_D = v_d \bullet \frac{d^2}{D^2} = 1.5 \bullet \frac{m}{s} \bullet \frac{(3\ mm)^2}{(20\ mm)^2} = 1.5\,\frac{m}{s} \bullet \frac{9\ mm^2}{400\ mm^2} = 0.03375\,\frac{m}{s}$$

Very high pressures are often used in hydraulics. For example some systems have a pressure of between 1 bar and 1,000 bar. As you can imagine, a fluid under high pressure has a large amount of energy. We call this **pressure energy**. This depends not only on the pressure, but also on the volume of the fluid. There is the following relationship:

pressure energy W = p • V $[W] = [p] \bullet [V] = \frac{N}{m^2} \bullet m^3$ = **Nm = J**

Note:	The pressure energy of a fluid is the product of the pressure *p* and volume *V*.

So far we have dealt with static liquids. Figure 3.5 takes a look at dynamic systems with liquids. In this case it is the flow of a liquid through a narrowing pipe:

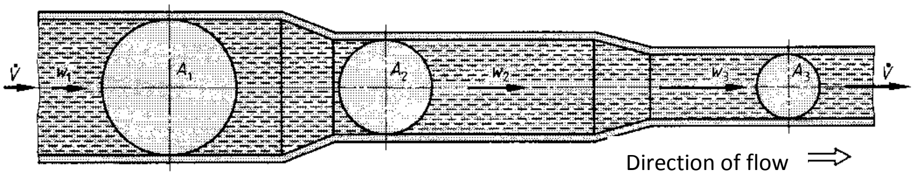


Figure 3.5: A liquid flowing

We can see the **cross sections of the flow** A_1, A_2 and A_3, and the flow speeds. We use the symbol *w* here for the speed rather than *v* as we do for solids.
We use the symbol $\dot{V}$ to denote the flow rate.

flow rate $\dot{V} = \frac{V}{t}$ $\rightarrow [\dot{V}] = \frac{[V]}{[t]} = \frac{m^3}{s}$

Note:	The term *flow rate* is used to describe the volume of a fluid flowing through a pipe in a certain time. It is the product of the flow speed w and the cross section of the pipe *A*.

flow rate $\dot{V} = w_1 \bullet A_1 = w_2 \bullet A_2$ = constant….. (flow equation)

The flow equation says:

Note:	The flow rate in a closed pipe is constant for an incompressible fluid.

Example 3.3:
A pipe has an internal diameter d_1 = 50 mm. An incompressible fluid is flowing in it with a flow speed of w_1 = 4 m/s. The pipe narrows to d_2 = 30 mm. What is the flow speed w_2 in the narrower part of the pipe?

Solution:

$$w_1 \bullet A_1 = w_2 \bullet A_2 \rightarrow w_2 = w_1 \bullet \frac{A_1}{A_2} = w_1 \bullet \frac{\frac{\pi}{4} \bullet d_1^2}{\frac{\pi}{4} \bullet d_2^2} = w_1 \bullet \frac{d_1^2}{d_2^2} = 4\,\frac{m}{s} \bullet \frac{(5\ cm)^2}{(3\ cm)^2} = 11.11\,\frac{m}{s}$$

We already know the relationship between energy and power from the dynamics of solids: *P = W/t*. If we put the pressure energy *W = p • V* into this equation and note that $\dot{V}$ = *V/t*, then we have:

hydraulic power P = p • $\dot{V}$ $\rightarrow [P] = [p] \cdot [\dot{V}] = \frac{N}{m^2} \cdot \frac{m^3}{s} = \frac{Nm}{s} = \frac{Ws}{s} =$ **W**

Example 3.4:
There is a pipe with an internal diameter of d_i = 200 mm. Water is flowing in this pipe at a rate of 15 m^3 per minute. The total pressure in the pipe is *p* = 8 bar. What is the hydraulic power?

Solution:
$P = p \cdot \dot{V} = p \cdot \frac{V}{t} = 800{,}000 \frac{N}{m^2} \cdot \frac{15\ m^2}{60\ s} = 200{,}000\ \frac{Nm}{s} = 200{,}00\ W = 200\ kW$

3.3 The important principles of aeromechanics (pneumatics)

We will look at the characteristics of gases in this unit. Gases are compressible – in contrast to liquids. This means that the volume of a gas depends heavily on the pressure. In addition, the volume is heavily influenced by the temperature. Thus we will start of by looking at the concepts of **air pressure** (in general, gas pressure) and **temperature**.

In every day life we do not need to think about the pressure of the air around us as we are used to living surrounded by air. We do know, however, that sometimes this air pressure is higher and sometimes lower. We can measure this with a barometer. The amount it changes is called the **margin of fluctuation**. In a space where there is no air, which we call a vacuum, there is, of course, no pressure. When one measures pressure relative to a vacuum, then say it is absolute pressure and use the symbol p_{abs}. In **DIN 1314** "Pressure" we have:

Note:	The *absolute pressure* p_{abs} is the pressure relative to a vacuum.

As we mentioned, the ambient air around us has a pressure of about 1 bar (we use the symbol p_{amb}). It is, however, sometimes a little higher and sometimes a little lower.

Note:	If we measure pressure relative to the prevailing air pressure then we call this gauge pressure and use the symbol p_e.

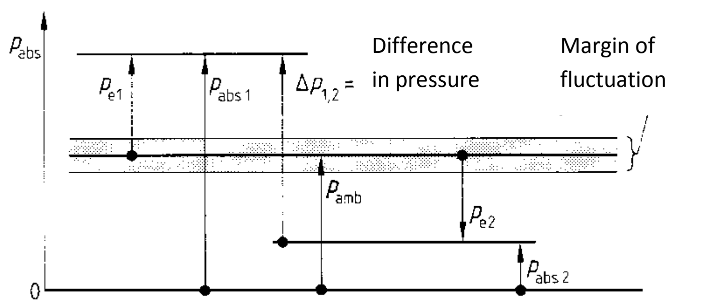


Figure 3.6: Absolute pressure p_{abs}, air pressure p_{amb}, gauge pressure p_e

From figure 3.6 we can see that:

gauge pressure $p_e = p_{abs} - p_{amb}$ This equation shows us that:

gauge pressure can be positive or negative. If the latter is the case, we may call it **low pressure,** as the absolute pressure is smaller than the **atmospheric pressure.**

Example 3.5:
We use the constant **standard atmosphere**. This has a value of p_n = 1.01325 bar. What is the absolute pressure in a container when the atmospheric pressure p_{amb} equals the standard atmosphere and when the a gauge pressure is p_e = 3?

Solution:
$p_e = p_{abs} - p_{amb}$ $\rightarrow p_{abs} = p_e + p_{amb}$ = 3 bar + 1.01325 bar = 4.01325 bar

There is a smallest possible temperature, it is called absolute zero. It is exactly –273.15 °C.

Note:	The temperature relative to absolute zero is called the absolute temperature. The symbol is *T*, and the unit is the Kelvin (K).

Normally we give temperatures relative to the freezing point of water. This is zero degrees Celsius (0 °C). We call such temperatures **Celsius temperatures**. The relationship between absolute temperatures and Celsius temperatures, given the symbol ϑ (theta), is shown in figure 3.7:

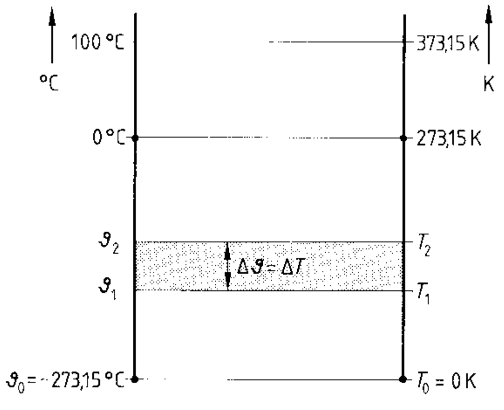


Figure 3.7: The relationship between absolute temperatures and Celsius temperatures

The relationship between T and ϑ $T = \vartheta + 273.15$ in K
$\vartheta = T - 273.15$ in °C

Figure 3.7 also shows that it does not matter if we give differences in temperature in K or °C.

Example 3.6:

Some steam is heated from ϑ_1 = 125 °C to ϑ_2 = 810 °C. Calculate

a) T_1 and T_2,

b) $\Delta\,\vartheta$ and ΔT.

Solution:

a) T_1 = (ϑ_1 + 273.15) K = (125 + 273.15) K = 398.15 K
T_2 = (ϑ_2 + 273.15) K = (810 + 273.15) K = 1,083.15 K

b) $\Delta\,\vartheta = \vartheta_2 - \vartheta_1$ = 810 °C – 125 °C = 685 °C
$\Delta T = T_2 - T_1$ = 1,083 K – 398.15 K = 685 K

So we can see: $\Delta\,\vartheta$ in °C = ΔT in K

There is a correlation between **temperature T, volume *V*** and **pressure *p*** for gases.
The relationship between *T*, *V* and *p* has been shown repeatedly in experiments. It is referred to as the **combined gas law**:

Note:	If the absolute state variables *T*, *V* and *p* change for a fixed mass of gas then the quotient of (*p* • *V*) and *T* is constant.

So we can write:

the combined gas law $\frac{p_1 \bullet V_1}{T_1} = \frac{p_2 \bullet V_2}{T_2}$ **when *p* and *T* are absolute values!**

There is a **very important rule** for all gas laws:

Note:	Pressure and temperature have to be given as *absolute values*.

Example 3.7:

There is a closed vessel of 3 m^3 volume. The air inside is at a temperature of 17 °C and the gauge pressure p_e= 2 bar. A piston forces the air into a space 20% smaller and the air is warmed to 100 °C at the same time. What is the absolute air pressure in the vessel when the atmospheric pressure p_{amb} = 1.01 bar?

Solution:

$\frac{p_{abs1} \bullet V_1}{T_1} = \frac{p_{abs2} \bullet V_2}{T_2}$

$p_{abs1} = p_{amb} + p_e$ = 1.0 bar + 2 bar = 3.01 bar

T_1 = (ϑ_1 + 273.15) K = (17 + 273.15) K = 290.15 K

$p_{abs2} = p_{abs1} \bullet \frac{V_1}{V_2} \bullet \frac{T_2}{T_1}$

T_2 = (ϑ_1 + 273.15) K = (100 + 273.15) K = 373.15 K

$V_1 = 3\,m^3$, $V_2 = V_1 - 0.2 \bullet V_1 = 3\,m^3 - 0.6\,m^3 = 2.4\,m^3$

p_{abs2} = 3.01 bar • $\frac{3\,m^3}{2.4\,m^3} \bullet \frac{373.15\ \text{K}}{290.15\ \text{K}}$ = 4.84 bar

The combined gas law is also able to make statements about the **air consumption in pneumatic systems**. If we call this V_2 then we have

air consumption $V_2 = V_1 \bullet \frac{p_1}{p_2} \bullet \frac{T_2}{T_1}$ in m^3 Index 1: Air supply

Index 2: Air used

In many cases, one of the three quantities is constant. Then we can simplify the combined gas law:

T = constant	→	$T_1 = T_2$	→	$p_1 \bullet V_1 = p_2 \bullet V_2$	**Boyle-Mariotte´s law**
p = constant	→	$p_1 = p_2$	→	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$	**Gay-Lussac´s first law**
V = constant	→	$V_1 = V_2$	→	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	**Gay-Lussac´s second law**

Example 3.8:
The air pressure in a container of pressurized air with *V* = 600 l is p_{abs1} = 12 bar. A valve opens and air flows out until the pressure is the atmospheric pressure p_{amb} = 1.02 bar. How many liters of air are left in the container if the temperature stays constant?

Solution:

$$p_{abs1} \bullet V_1 = p_{abs2} \bullet V_2 \rightarrow V_2 = V_1 \bullet \frac{p_{abs1}}{p_{abs2}} = 600\ \text{l} \bullet \frac{12\ bar}{1.02\ bar} = 7058.81\ \text{l}$$

3.4 Electricity

Now we have considered mechanical, hydraulic and pneumatic transmission, we will focus on the important laws of **electrical power transmission**. These form the basis for both electrical open-loop and closed-loop control systems.

3.4.1 Electric current – electric potential

As we saw in table 3.1, electric transmission is carried out by free electrons in the structure of the metal. This is shown in figure 3.8:

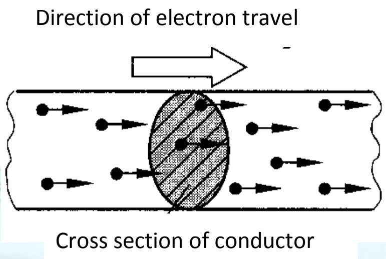


Figure 3.8: Electric current

Metals are particularly good **conductors** of electricity but carbon, acids, bases and salts in solution also conduct electric current. Under certain conditions, non-conductors can still conduct electricity, for example gases in gas-discharge lamps.

Table 3.3: Electric conductors and non-conductor (insulator)

Conductors	Metals, carbon, acids, bases, salt solutions, gases (under certain conditions)
Non-conductors	Textiles, plastics, glass, amber, porcelain, vacuum, oil etc.

Note: Electric current in a conductor is the movement of free electrons in a particular direction. It has symbol I and is measured in Ampere (A).

Electric current is a base unit. Two electric currents exert a force on each other. This was used in the 9th general conference for measurements and weights in l948 to decide that:

Note: An ampere is defined to be the constant current that will produce an attractive force of 2×10^{-7} Newton per metre of length between two straight, parallel conductors of infinite length and negligible circular cross section placed one metre apart in a vacuum.

As you can see, there is a lot of theory in this definition and it is not really of practical use. However, we can see that electric current has the potential to create force.

A liquid flowing in a pipe has to overcome resistance, electrons have to do the same when flowing in a conductor. It is called electric resistance and given the symbol *R*. Figure 3.9 shows a systematic representation of what is known as an **electric circuit**:

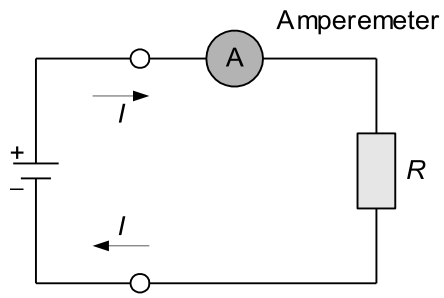


Figure 3.9: Simple electric circuit

Such a systematic diagram of an electric circuit (figure 3.9) is called a **circuit diagram.** The current is measured by an **ammeter**. It is easy to show that a current can only flow when the circuit is closed i.e. when there is not gap in the connections. A closed circuit has to consist of at least a power source, a **consumer** (in figure 3.9 the resistance *R*) and a **feed line** and **return line.** The power source in figure 3.9 supplies **direct current** with a **positive pole** and a **negative pole.**

negative pole -> place with **excess electrons**
positive pole -> place with **lack of electrons**

} Electrons flow from the negative pole to the positive pole

The **direction of the current** in figure 3.9 is from + to –, whereas the electrons flow the other direction. This is because it was decided which direction the flow would be termed before electrons were discovered.

Note:	The direction of the current is opposite to the direction of travel of the electrons.

Another form of current is **alternating current (AC)**. It is different to **direct current (DC)**. If a direct current and an alternating current are mixed, we get an **undulatory current.**

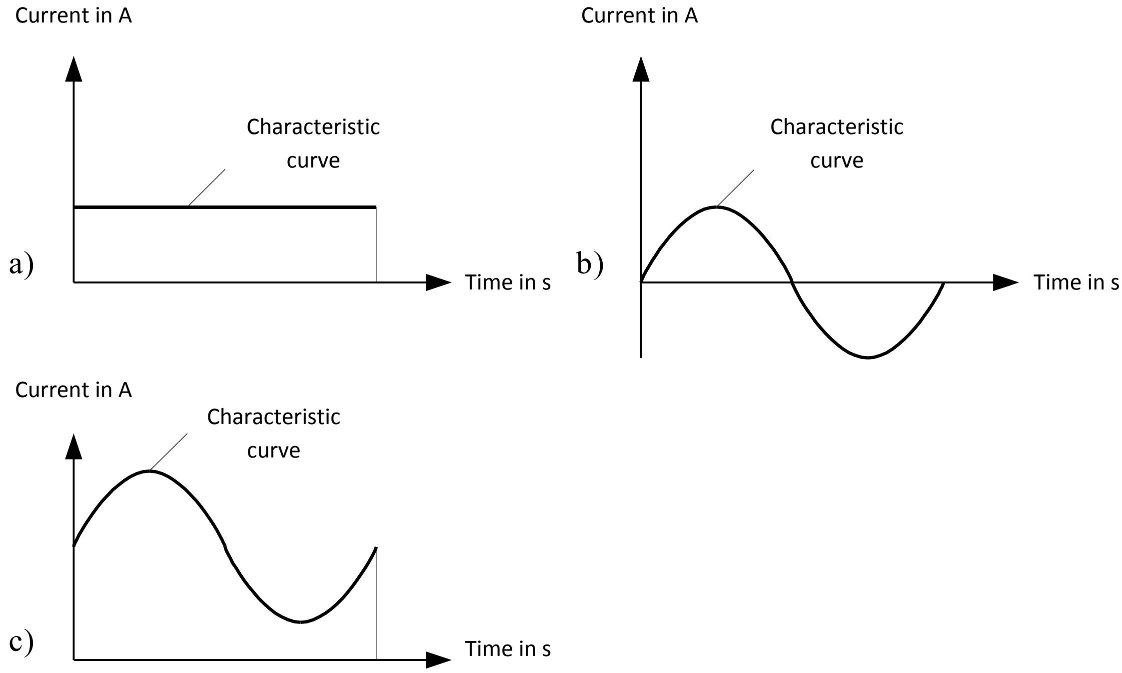


Figure 3.10: a) direct current, b) alternating current, c) undulatory current

In practice, an alternating current made from adding three separate alternating currents is often used, this is called **three-phase electric current.**
We will now consider the terms excess of electrons and lack of electrons. Here is a reminder from chemistry:

Note:	*Electrons* are the negatively charge parts of the atomic shell. Each carries a charge, the smallest negative charge, the *elementary electric charge.*

Thus wherever there is an excess of electrons there is a **negative electric charge** and wherever there is a lack of electrons, there is a **positive electric charge**. Thus there is a difference in charge at the terminals of the power source, which we call potential. The symbol used is U, and the unit is the Volt (V). DIN 1301 says:

Note:	The *electric potential* U is the quotient of the work W necessary for the electric flow and the total charge transported Q.

The unit of charge is the **Coulomb** (C), and is related to electric current via time:

electric current $I = \frac{Q}{t}$ and

electric charge $Q = I \bullet t$ $[Q] = [I] \bullet [t] = A \bullet s = \mathbf{As}$ $\rightarrow 1\ C = 1\ As$

According to the definition for electric potential, we have

electric potential $U = \frac{W}{Q}$ $[U] = \frac{[W]}{[Q]} = \frac{Nm}{As} = \frac{Nm}{C} = \frac{J}{C} = \mathbf{Volt}$

Example 3.9:

A car battery has a capacity (charge) of Q = 24 Ah (Amp hours). How much is this in as and how long can a current of 1.2 A flow?

Solution:

$Q = 24\ Ah = 24\ Ah \bullet 3{,}600\ \frac{s}{h} = 86{,}400\ As$; $I = \frac{Q}{t} \rightarrow t = \frac{Q}{I} = \frac{86{,}400\ As}{1.2\ A} = 72{,}000\ s = 20\ h$

3.4.2 Electric resistance

The most important law about electricity is **Ohm's law.** It links the current, potential and resistance. The physicist Georg Simon Ohm determined in 1827 that:

Note:	The current is proportional to the potential and inversely proportional to the resistance. The resistance is measured in Ohms (Ω).

Ohm's law $I = \frac{U}{R}$ in A $\rightarrow [I] = \frac{[U]}{[R]} = \frac{Volt}{Ohm} = \frac{\mathbf{V}}{\mathbf{\Omega}}$ $\rightarrow 1\ A = \frac{1\ V}{1\ \Omega}$

We can rearrange this equation to get:

electric potential $U = R \bullet I$ in V $\rightarrow 1\ V = 1\ \Omega \bullet 1\ A$

electric resistance $R = \frac{U}{I}$ in Ω $\rightarrow 1\ \Omega = \frac{1\ V}{1\ A}$

Example 3.10:

Calculate the current passing through a resistor of electric resistance R = 6 Ω in Amps and in Milliamp when there is a potential of U = 15 V across it.

Solution:

$I = \frac{U}{R} = \frac{15\ V}{15\ \Omega} = 2.5\ A = 2{,}500\ mA$

The electrical resistance of a resistor like a metal wire depends, of course, on the material from which it is made as some materials conduct better than others. We use the term **electrical resistivity** and the symbol ρ. Of course the length of the conductor and its cross section *A* are also important. There are analogies here with the flow of liquids too. If one doubles the length of a conductor, then one doubles the resistance. For the cross section, we have the opposite: If the cross section *A* is doubled, the resistance is halved as the electrons have much more space. This leads to the definition of the

resistance of a conductor $R = \rho \bullet \frac{l}{A}$ in Ω. So

electrical resistivity $\rho = \frac{R \bullet A}{l}$ $\rightarrow [\rho] = \frac{[R] \bullet [A]}{[l]} = \frac{\mathbf{\Omega \bullet mm^2}}{\boldsymbol{m}}$

Note:	When calculating electric resistance, the length is measured in m and the cross section is mm^2.

Both the electric resistivity and so also the **resistance** are **temperature dependent**. Table 3.4 shows some values.

Table 3.4: Electric resistivity at 20 °C

Material	**Al**	**Cr**	**Fe**	**Cu**	**Ni**	**Steel**	**Ti**	**V**	**Zn**	**Sn**
resistivity in $\Omega \bullet \mathbf{mm^2/m}$	0.0778	0.13	0.1	0.0178	0.095	0.13	0.8	0.2	0.0625	0.115

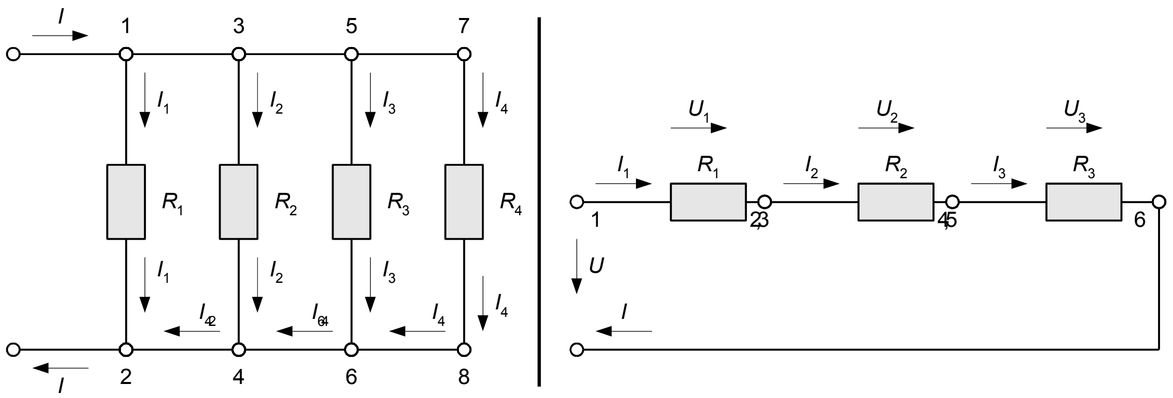
Example 3.11:

A copper conductor (ρ = 0.0178 $\Omega \cdot \mathrm{mm^2/m}$) has a cross section of A = 1.5 $\mathrm{mm^2}$. When a potential of U = 230 V is applied, a current of I = 6 A flows. Calculate *l*, the length of the conductor.

Solution:

$$R = \frac{U}{I} = \rho \bullet \frac{l}{A} \quad \rightarrow l = \frac{U \bullet A}{\rho \bullet I} = \frac{230\,V \bullet 1.5\,\mathrm{mm^2}}{0.0178\,\Omega \bullet \frac{\mathrm{mm^2}}{m} \bullet 6\,A} = 3{,}230\,\frac{\Omega}{\frac{\Omega}{\mathrm{m}}} = 3{,}230\text{ m}$$

When more than one resistor is in an electric circuit, we have to distinguish whether they are connected in **parallel** or in **series.** The difference is shown in figure 3.11:



a) Parallel connection b) Series connection

Figure 3.11: Types of electric connection

We can see from figure 3.11 that the current forks in a parallel connection at the **nodes** (1, 2, 3,...) and the total current is the sum of the currents passing through the individual parts. The current does not branch like this in a series connection. In a series connection, all the current passes through each of the resistors. We have, for each node:

Kirchhoff's current law $\sum I_{hin} = \sum I_{ab}$

Kirchhoff's current law allows us to calculate equations for both types of connection. These equations can partly be seen directly using figure 3.11. We do not have space to show the derivation of these formulae here. The most important equations are given in Table 3.5:

Table 3.5: Formulae for parallel and series connections

	Parallel	**Series**
Total current	$I = I_1 + I_2 + I_3 + \ldots$	$I = I_1 = I_2 = I_3 = \ldots$
Total potential	$U = U_1 = U_2 = U_3 = \ldots$	$U = U_1 + U_2 + U_3 = \ldots$
Total resistance	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$	$R = R_1 + R_2 + R_3 = \ldots$

Example 3.12:

Two resistors, $R_1 = 3\ \Omega$ and $R_2 = 5\ \Omega$ are connected in parallel.

a) Find an equation which gives the total resistance *R*.

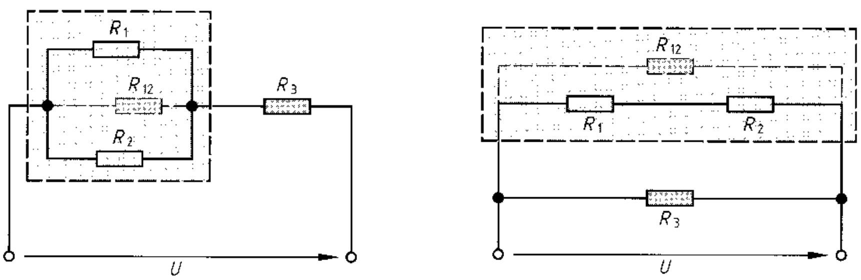
b) What is the total resistance *R*?

Solution:

a) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow \frac{1}{R} = \frac{R_2}{R_1 \cdot R_2} + \frac{R_1}{R_1 \cdot R_2} = \frac{R_1 + R_2}{R_1 \cdot R_2} \rightarrow R = \frac{R_1 \cdot R_2}{R_1 + R_2}$

b) $R = \frac{3\ \Omega \cdot 5\ \Omega}{3\ \Omega + 5\ \Omega} = \frac{15\ \Omega^2}{8\ \Omega} = 1.875\ \Omega$

The laws for connecting in parallel and in series are valid for arbitrarily many resistors and can also be used when resistors are connected in a **mixed** way. We can classify these situations as **extended series connections** or **extended parallel connections**. See figure 3.12b.



a) Extended series connection

b) Extended parallel connection

Figure 3.12: Mixed connection of resistances

As you can see in figure 3.12, we first collect together the resistors R_1 and R_2, thinking of them as a single resistance R_{12}. This technique allows us simplify calculations for complicated connections. We get the following result for the situations in figure 3.12.

Total resistance for the extended series connection $R = R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2}$ in Ω

Total resistance for the extended parallel connection $R = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3}$ in Ω

Example 3.13:

The resistors shown in figure 3.12a have resistance R_1 = 10 Ω, R_2 = 30 Ω, R_3 = 42 Ω. The potential applied is U = 125 V. Calculate

a) the total resistance R,

b) the total current *I*

Solution:

a) $R = R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2} = 42\ \Omega + \frac{10\ \Omega \cdot 30\ \Omega}{10\ \Omega + 30\ \Omega} = 42\ \Omega + 7.5\ \Omega = 49.5\ \Omega$

b) $I = \frac{U}{R} = \frac{125\ V}{49.5\ \Omega} = 2.525\ A$

Rearranging the defining equation for electric potential $U = \frac{W}{Q}$ we get:

3.4.3 Electrical work and power in direct current circuits

electric work $W = U \cdot Q$ $\rightarrow [W] = [U] \cdot [Q] = V \cdot C = \frac{Nm}{C} \cdot C = \mathbf{Nm = J}$

Electric work and, of course, electric energy have the same unit as mechanical energy, Nm. As mentioned earlier, this is equivalent to other units:

Equivalence of units for energy 1 Nm = 1 J = Ws

You will also know that when talking about electricity, one uses the **Watt second** (Ws) or for larger amounts the **kilowatt hour** (kWh).

1 kWh $= 1\ kWh \cdot 3{,}600\ \frac{s}{h} \cdot 1{,}000\ \frac{W}{kW} =$ **3,600,000 Ws**

Electrical power is the energy or work done over the time, just like for mechanical power (and any other type). Thus as $Q = I \cdot t$, we have

electric power $P = \frac{W}{t} = \frac{U \cdot Q}{t} = \frac{U \cdot I \cdot t}{t} = U \cdot I$ $\rightarrow [P] = [U] \cdot [I] = \mathbf{V \cdot A}$

According to DIN 1304, the product V • A can be replaced by the unit W (Watt).

unit of electric power 1 Volt-ampere = 1 Watt → 1 VA = 1 W

Using Ohm's law, it is possible to substitute $I = U/R$ for the current or $R \bullet I$ for the potential. This generates the following equations:

electric power $P = U \bullet I = \frac{U^2}{R} = R \bullet I^2$ in W

Example 3.14:
The manufacturer of a consumer gives the following **nominal values**:
P_n = 200 W, U_N= 230 V (230 V/200 W for short). Calculate
a) the resistance *R* of the consumer,
b) the current.

Solution:

a) $P = \frac{U^2}{R}$ $\rightarrow R = \frac{U^2}{P} = \frac{(230\text{ V})^2}{200\text{ W}} = 264.5\ \Omega$

b) $P = U \bullet I$ $\rightarrow I = \frac{P}{U} = \frac{200\text{ W}}{230\text{ V}} = 0.87$ A
Test: $P = R \bullet I^2 = 264.5\ \Omega \bullet (0.87\text{ A})^2 = 200$ W

3.5 Measuring electrical values

We have already looked at the instrument that measures current, the **ammeter.** There is also an instrument to measure the potential, the **voltmeter**. We have also looked at the difference between parallel and series connections. Look at figure 3.11 again. From this picture and the formulae in table 3.5, especially those for

Total current for a series connection $I = I_1 = I_2 = I_3 = ...$ and
Total potential for a parallel connection $U = U_1 = U_2 = U_3 = ...$
we can see

Note:	An ammeter must be connected in series to the consumer and power source.

***A voltmeter must be connected in parallel* to the consumer and power source.**
These connections are shown in figure 3.13:

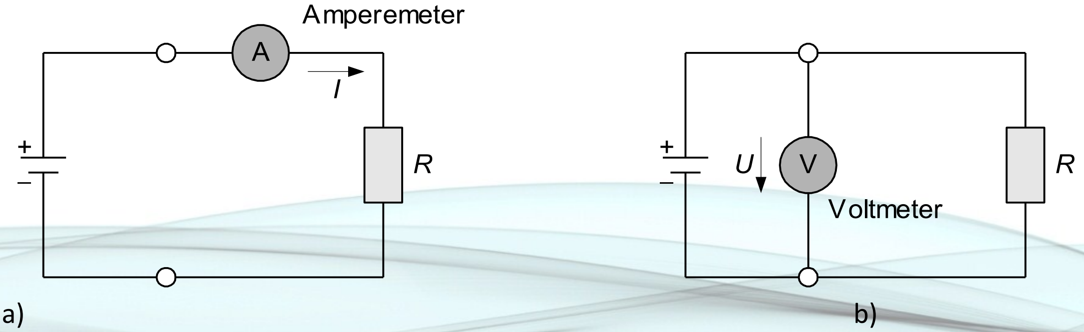


Figure 3.13:
a) Ammeter connected in series b) Voltmeter connected in parallel

As it is possible to measure the current and the potential, one can indirectly measure the power as

electric power P = U • I → $[P] = [U] \cdot [I]$ = V • A **= W**

It is also possible to measure electric power directly using a **resistance bridge**. However, we do not cover that here.

Example 3.15:

a) Sketch the circuit diagram for measuring **electric power indirectly.**

b) What is the electric power in kW if the potential is U = 150 V and the current is I = 65 A?

Solution:

a)

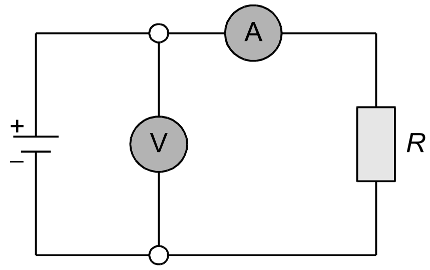


Figure 3.14: Measuring power indirectly

b) $P = U \cdot I$ = 150 V • 65 A = 9,750 VA = 9,750 W = 9.75 kW

It is also possible to use **electric measuring instruments** to measure non-electric values. Their **sensors,** change the non-electric values into electric values. Table 3.6 has some examples.

Table 3.6: Properties and the sensors that measure them

Property	**Sensor**	**Transformation into electric property**
Temperature	temperature dependent resistor	$\Delta\vartheta \rightarrow \Delta R$
Pressure	piezoelectric crystal	$\Delta\vartheta \rightarrow \Delta U$
Heat energy	thermocouple	$\Delta\vartheta \sim \Delta Q \rightarrow \Delta U$
Light intensity (power)	photovoltaic cell	$\Delta P \rightarrow \Delta U$

3.6 Alternating current, three-phase current

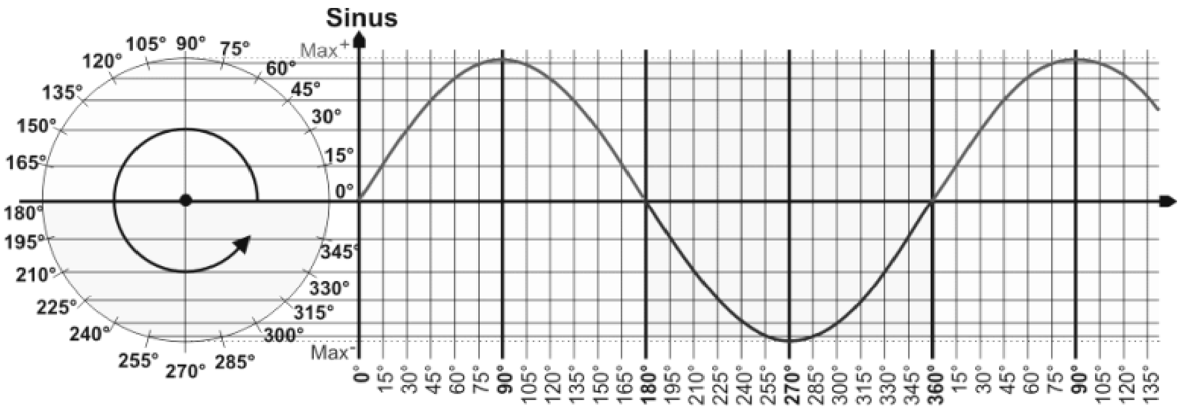
3.6.1 Alternating current

Alternating current (AC) is electric current which changes its direction periodically.
There are many types, but the most common is "sinusoidal alternating current".

Alternating current is created when a loop of wire keeps turning in a magnetic field. Each side of the loop goes to the left and then the right of the magnetic field. This creates increasing and then decreasing potential going in one direction and then the other.

Generators are used in industry to create alternating currents. Instead of a single loop of wire, coils with many turns are used and instead of a single pair of magnetic poles, many pairs are used. This allows the creation of a high enough potential and a large enough frequency.

Sinusoidal alternating current can also be generated from direct current using computer-controlled power electronics, e.g. by inverters to convert current from solar cells to that suitable for the mains.



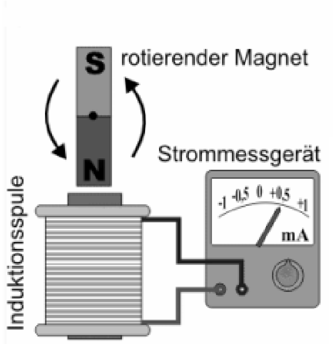


Figure 3.15: The generation of sinusoidal alternating current

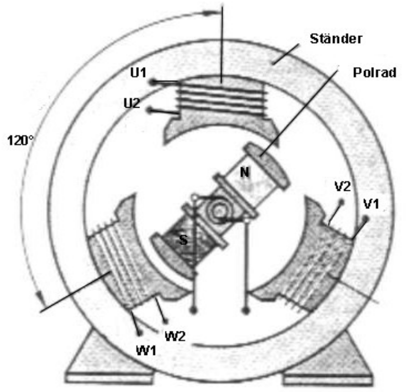
3.6.2 Three-phase current

Another type of alternating current is three-phase current.
In practice, instead of one alternating current, three separate alternating currents in different phases are created in the generator. These three phases are carried in different wires. Three-phase current is created by using three coils positioned regularly around the edge of the circle. The phase of the AC potential in the coils then differs by 120. The individual phases of this industrial alternating current can then be used individually. Three-phase current is made by linking the three potentials.

The "linking factor" for sinusoidal alternating current is $\sqrt{3}$.
Thus a three-phase current system makes 2 different potentials available:

the potential for each phase is 230 V so there are 3 • 230 V potentials
and the line to line voltage 3 • 400 V (= 3 • 230 V$\sqrt{3}$).

If one puts these potentials into a motor with three coils spaced out around a circle then a rotating magnetic field is created which drives the rotor of a three-phase motor.



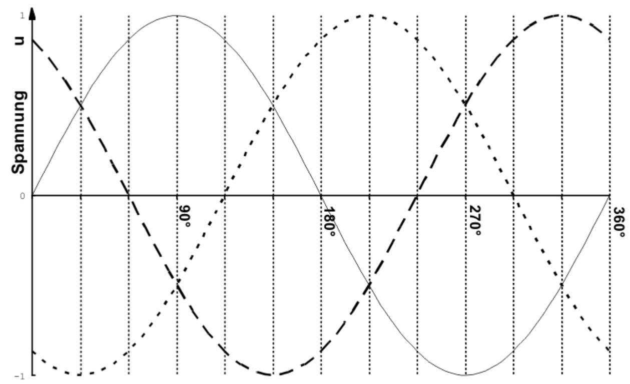


Figure: 3.16: The generation of three-phase current and the time dependence of the phases

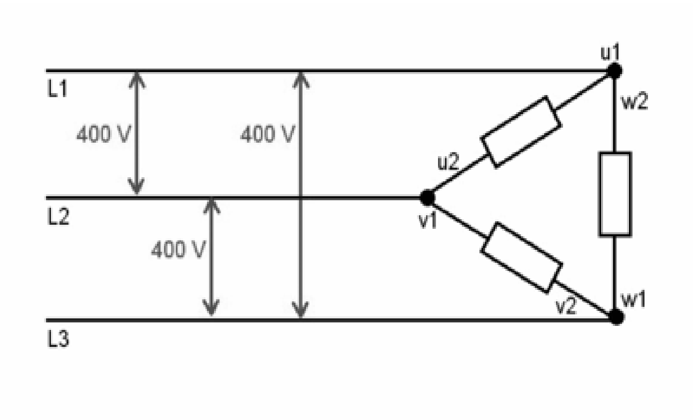


Figure 3.17: Delta connection

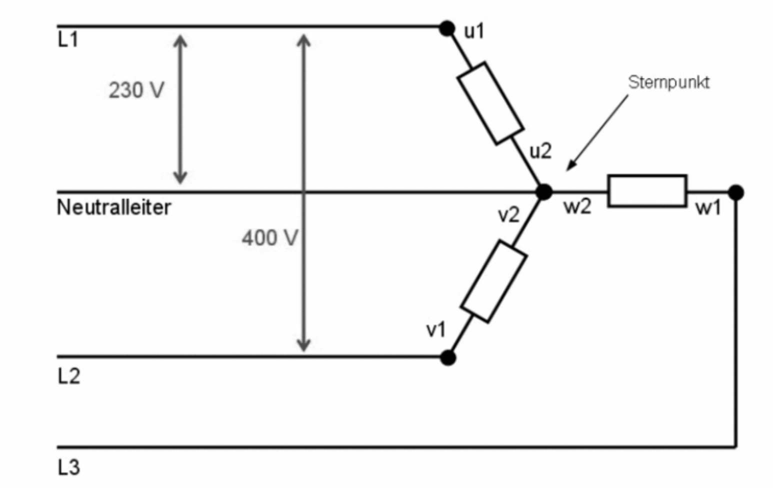


Figure: 3.18: Y-connection

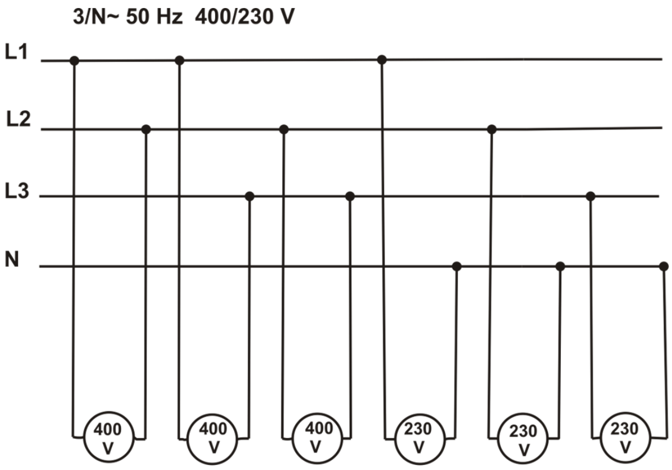


Figure: 3.19: The potentials in alternating and three-phase current

3.6.3 Frequency and period

The number of alternations in a second is called the frequency, it is measured in Hertz (Hz).

A period is the time taken for one cycle in a repeating event in a physical system. In alternating current this means that the current is positive and then negative and returns to the same value as at the start.

The period T is the inverse of the frequency f

T = 1/f [s]

In Germany alternating current generally has a period of: T = 1 / 50 Hz = 0.02 s

The frequency depends on the frequency at which the rotors in the generators rotate. The mains in Germany and other European states normally has a frequency of 50 Hertz (Hz). This means that, for example, the rotor of a generator with one pair of poles per phase must turn 3000 times a minute. If one has more pairs of poles, then the frequency of rotation is correspondingly lower.

The formula is:

$f = \frac{n}{p \cdot 60}$ (Hz)

f= frequency (Hz; s^{-1})
n= frequency of rotation (min^{-1})
p= number of pairs of poles

3.6.4 Root mean square

As the current and potential vary sinusoidally, calculating electrical quantities can be problematic. For example one can not simply calculate the power drain of a resistor with P=U*I. Which potential should we use in the equation if it keeps changing? We could calculate the power at any precise moment, but generally that's not what we are interested it. For this reason, we compare the effect it has with the effect a direct current would have. The root mean square of a phase voltage is the potential which a direct current would need to have the same effect.

Unless otherwise stated, when talking about alternating current / phase voltages, we will always mean the root mean square.
The peak value of a sinusoidal alternating current or phase voltage can be calculated with the following formula:

Peak value = $\sqrt{2}$ *** root mean square**

$\hat{u} = \sqrt{2} * U$

$\hat{i} = \sqrt{2} * I$

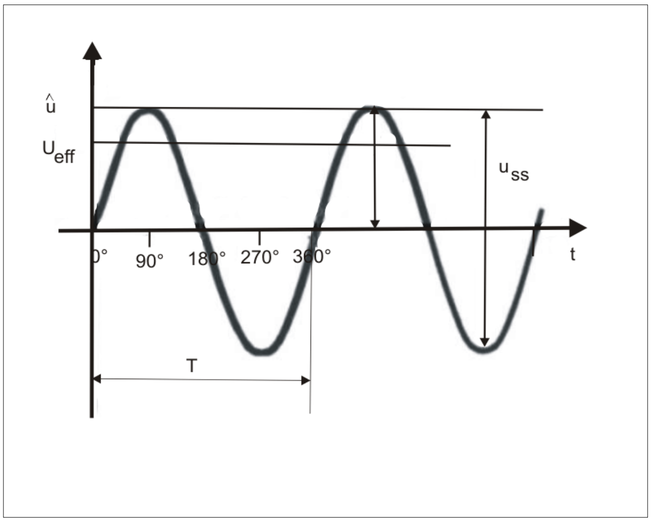


Figure: 3.20: Peak and root mean square values

3.6.5 Resistance and alternating currents

Every electric appliance has a resistance.

Depending on the type of appliance, there is

- Ohmic resistance (e.g. heating element)
- capacitive resistance (e.g. capacitor)
- inductive resistance (e.g. inductor)

Resistance is Ohmic is the current and potential have the same phase.

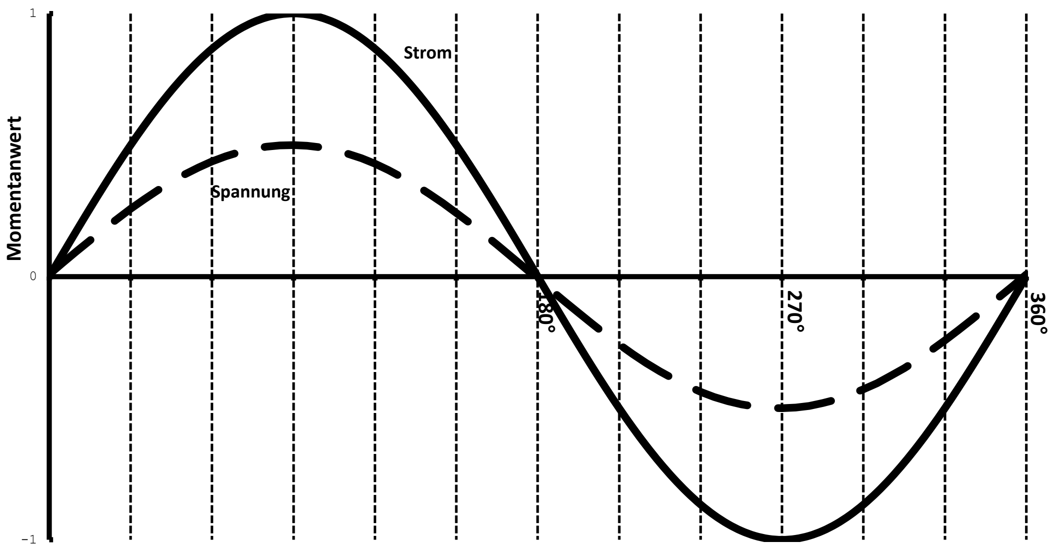
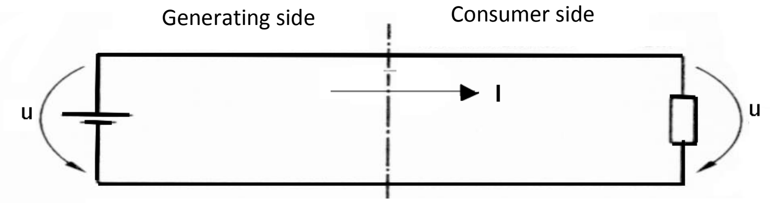


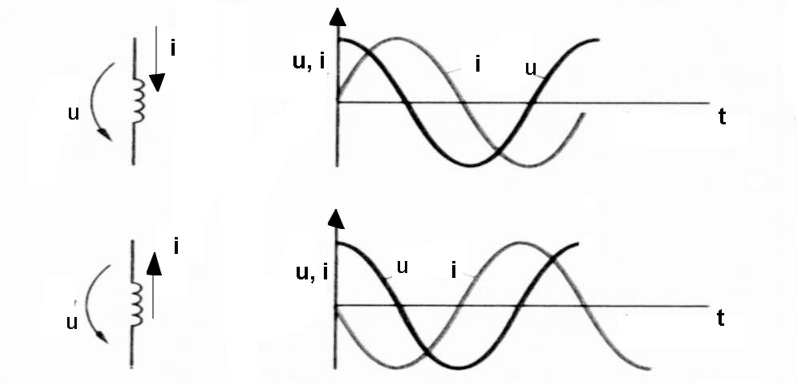
Figure: 3.21: Current and potential for an Ohmic resistance

Capacitive resistance and inductive resistance behave differently when the potential is varying than they do with direct current. They change the phase of the current so it is out of phase with the potential.

Note:	The following remarks are based on the voltage over the consumer.



Reference arrows fort he generating side and the consumer side



Current and voltage for induction
Above: as seen in the consumer side
Bottom: as seen in the generator side

Figure 3.22:

In a **consumer-arrow system** we draw the arrow for the current in the same direction as that for the potential.
In a **generator-arrow system** we write the arrow for the current in the opposite direction to that of the potential.

3.6.6 Capacitors in alternating currents

In direct current, current will only flow through a capacitor while it charges up. Then it breaks the electric current as between the capacitor plates there is an electric insulator.
In alternating current there is constantly current through a capacitor, due to the charge on its metal plates being constantly changed. The current is limited by the capacitive resistance **X_C = 1 / (ω*C)**.
Capacity is measured in Farads F [As/V],

ω = 2 *π*f is the angular frequency of the potential applied.

The current is 90° ahead, charging the capacitor and thus building up the potential on its plates.

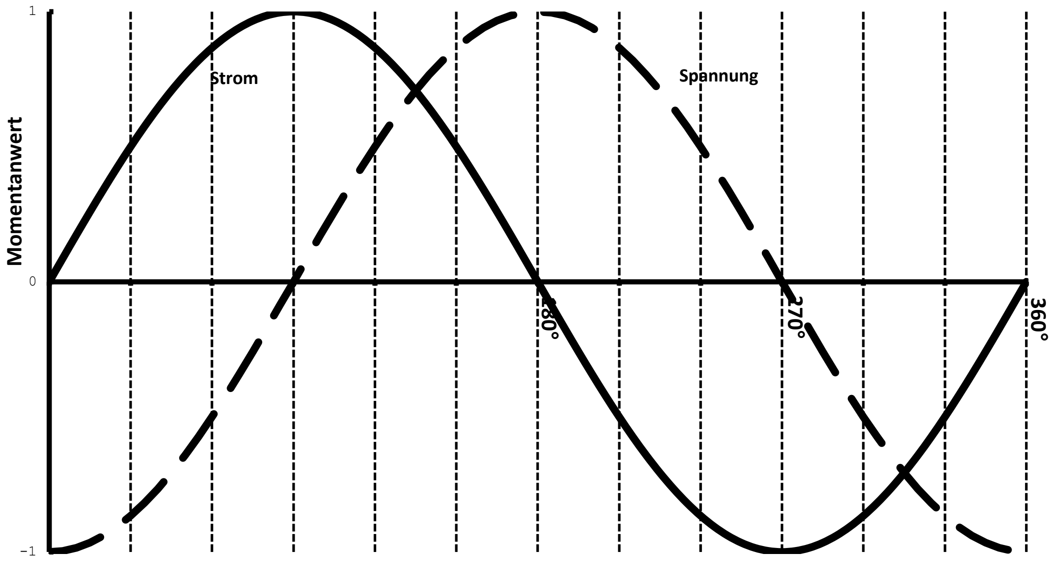


Figure 3.23: The current and potential for capacitive resistance

3.6.7 Inductors in alternating current

With a loss-free inductor, the potential is 90° ahead of the current as self-induction in the inductor creates a potential, which generates the current 90° later. The inductive resistance of the inductor is **X_L = ω*L**.

Induction is measured in Henrys H [Vs/A].

ω = 2* π *f is the angular frequency of the potential applied.

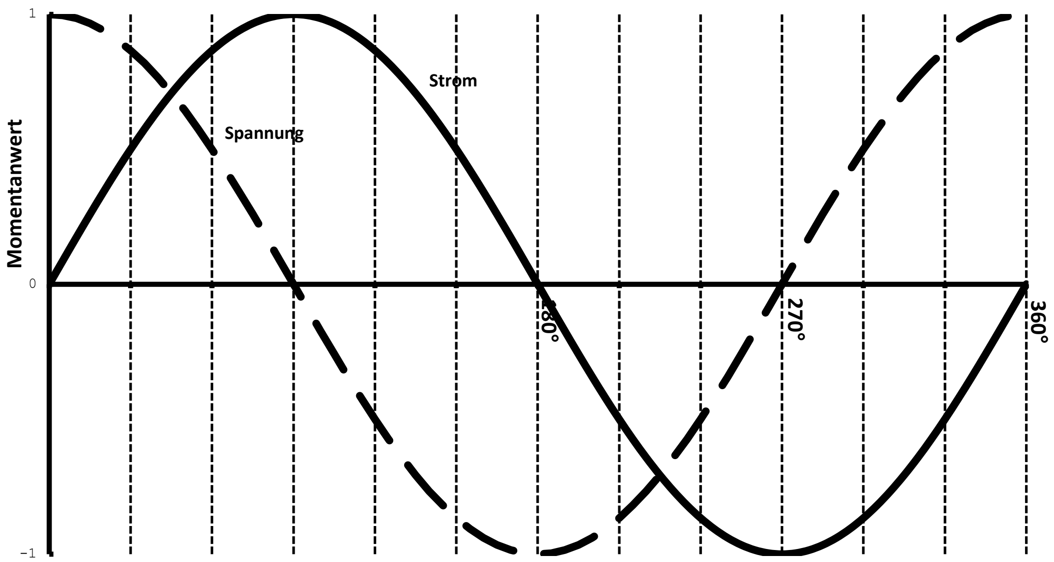


Figure 3.24: Current and voltage for inductive resistance

3.6.8 Active power, reactive power and apparent power for alternating currents

If a current I flows through an Ohmic resistance with constant potential U, then we can calculate the power as P: **P = U*I [W]**
Current and potential achieve their maximal values at the same time for an Ohmic resistance in alternating current and also have their zeros at the same time: They have the same phase.

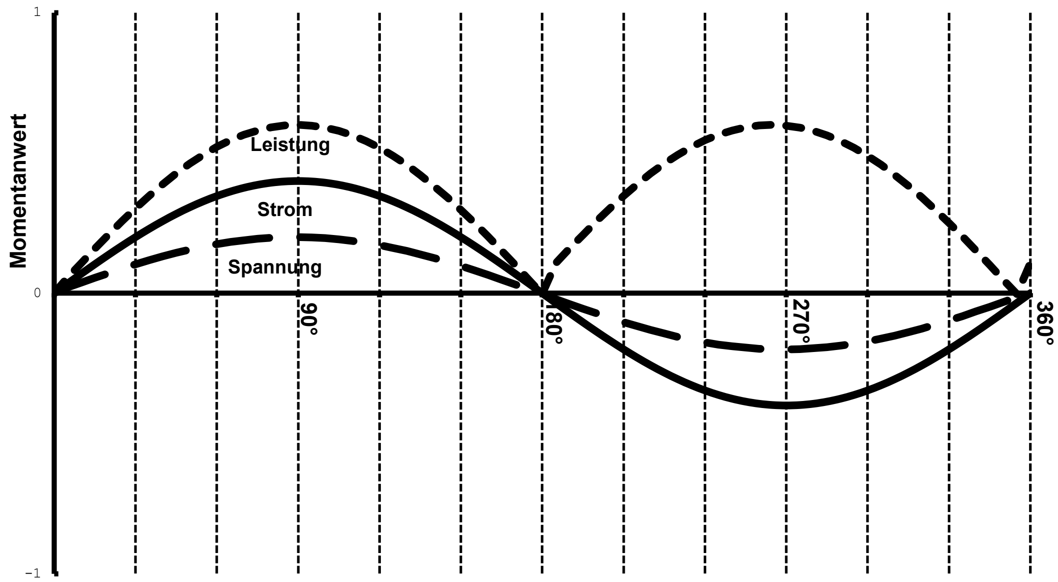


Figure 3.25: Current, voltage and power for Ohmic resistance

If an inductor or capacitor is connected then, due to the phase shift, capacitive or inductive reactive power is created.
The phase of the current is 90 degrees from that of the potential. Here, the reactive power is also shifted by 90 degrees to the active power.

The apparent power S can be calculated from the active power P and the reactive power Q with the following formula:

$$\mathbf{S = U_{eff} * I_{eff}} = \sqrt{P^2 + Q^2} \qquad \text{or } s = \sqrt{3} * U * I\ [VA]$$

Apparent power S is measured in VA (volt-ampere)
Reactive power Q is measured in var (volt-ampere reactive)
Active power P is measured in W (Watt)

3.6.9 Power factor

The term $\cos\varphi$ is known as the power factor. It is the quotient of the active and apparent power:

$\cos\varphi$ = P/S

For sinusoidal currents, the active power is:

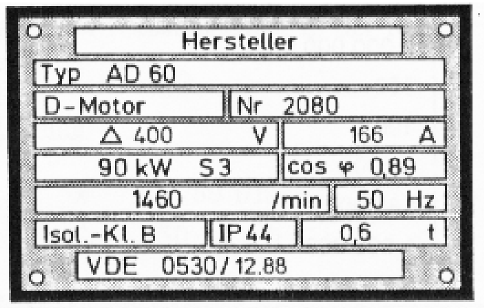
$P = U \cdot I \cdot \cos\varphi = \frac{1}{2} \cdot \hat{u} \cdot \hat{\imath} \cdot \cos\varphi$ [W]

The reactive power, which builds the t electric and magnetic fields in the circuit is given by:

$Q = U \cdot I \cdot \sin\varphi = S \cdot \sin\varphi = S \cdot \sqrt{1-\cos\varphi^2}$ [var]

Example 3.16:

An electric motor's rating plate has the following data:



Calculate the:

a) apparent power

b) active power

c) reactive power

Solution:

a) $S = \sqrt{3 \cdot U \cdot I} = \sqrt{3 \cdot 400\ V \cdot 166\ A}$ = 115 kVA

b) $P = S \cdot \cos\varphi$ = 115 kVA • 0.89 = 102.4 kW

c) $Q = S \cdot \sqrt{1-\cos\varphi^2}$ = 115 kVA$\sqrt{1-0.89^2}$ = 52.4 kvar

Note: The power on the rating plate of 90 kW is the power P_{ab} with which the motor drives the shaft. We can calculate the efficiency of the motor by using the active power P_{zu} we calculated in b):

$\eta = P_{ab} / P_{zu}$ = 90 kW / 102.4 kW = 0.88

3.7 Types of electric faults

3.7.1 Short-circuit

A short-circuit is a direct conducting path between two active electric poles, for example:

- between the positive and negative poles of a battery
- between the lines L1-L2 and/or L2-L3 and/or L3-L1 of a three-phase system
- between a line L1, L2 or L3 and the neutral wire or PEN conductor.

Short-circuits are generally caused by damaged insulation or a wrong connection in electric equipment or circuits.

While the electric potential goes down to almost zero, the current reaches its maximum, the short-circuit current.

DIN/VDE 0102 contains rules and regulations for calculating the short-circuit current of electric switchboards.

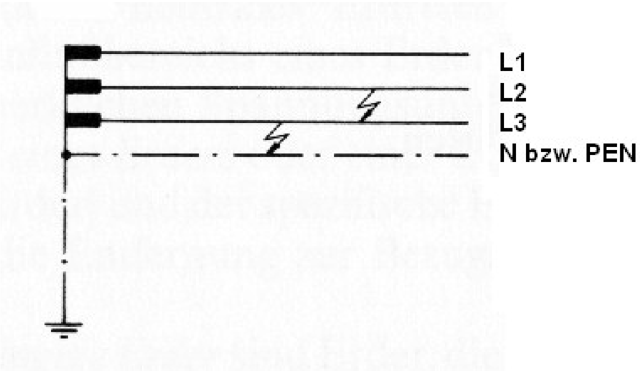


Figure 3.26: Short-circuit

3.7.2 Short to the housing

A **short to the housing** is when there is a problem (e.g. with insulations) leading to a connection between the live circuit and the conductive body (housing) of a piece of electrical equipment. The potential generated must not be more than 50 Volt AC or 120 Volt DC. If these values are exceeded, safety measures must be taken to isolate the equipment from the power source.

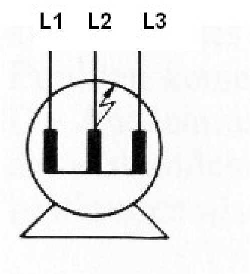


Figure 3.27: Short to the housing

3.7.3 Earth fault

An **earth fault** is a when there is a connection between the earth or an earthed part and a live line (e.g. L1; L2; L3;…) or neutral wire which is normally isolated.
As this causes a current in the earth, the potential caused in the earth close to the point of contact / live line gives a danger of electric shock for people and animals.
People or animals can, with a stride, span a distance in which the voltage drops. The difference between these two potentials is called the **step potential**. It is defined as the change in the earth's potential which can be spanned in 1m.

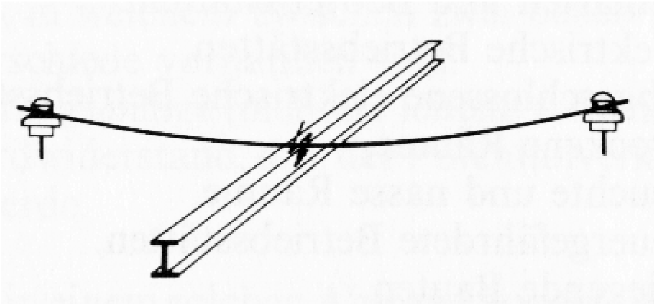


Figure 3.28: Earth fault

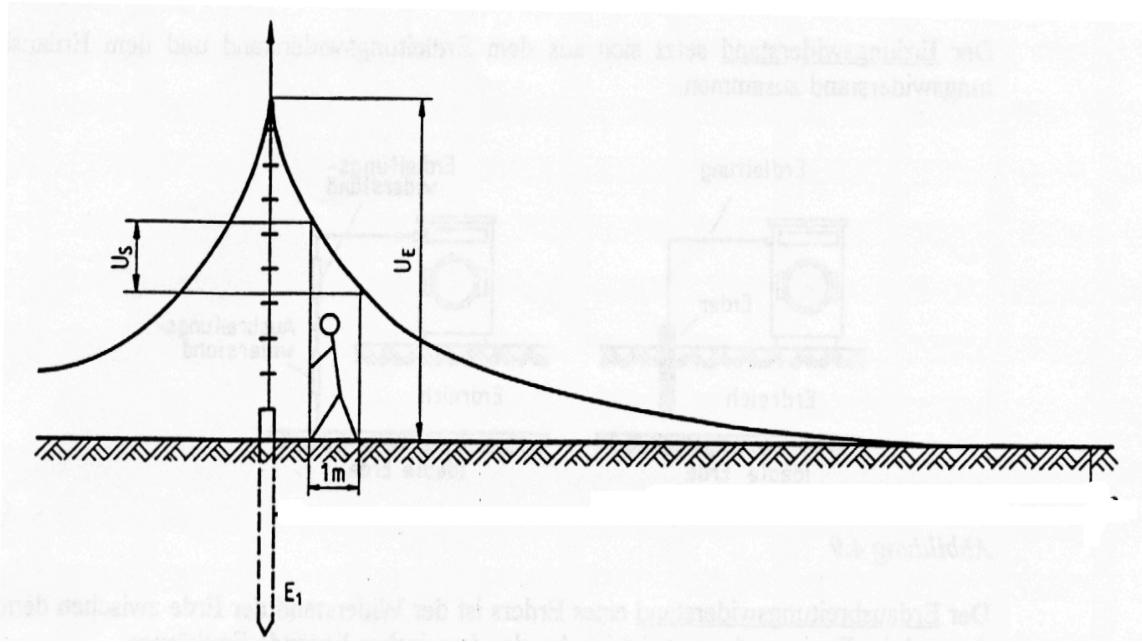


Figure 3.29: Earth potential, step potential

3.7.4 Conductor short-circuit

A **conductor short-circuit** is when there is an unintended electrical contact between a line and the neutral or PEN wire going via a resistance (load).

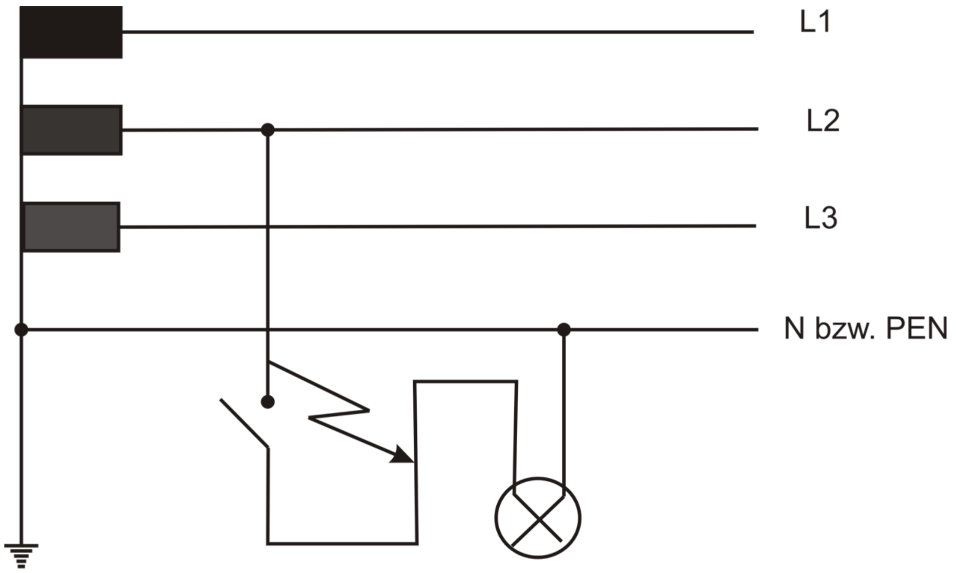


Figure 3.30: Conductor short-circuit

3.8 Overcurrent protection devices

Devices protecting against overcurrent protect conductors and equipment from thermal overload. Thermal overload occurs when there is overcurrent or a short-circuit. Circuit breakers or fuses protect wires and cables, and electrical equipment whereas motor circuit breakers protect electric machines. Motor circuit breakers are similar to circuit breaker. However, they are adapted to the demands of the equipment to be protected.

3.8.1 Circuit breakers

Circuit breakers have two separate trip mechanisms to protect against overload and short-circuits respectively.
To protect against overload there is a delayed-action thermal bimetallic trip.

To protect against short-circuits there is an electromagnetic trip which works almost immediately. All circuit breakers have a trip-free mechanism. The term "trip-free mechanism" means that the trip also works when the switch is held in the "on" position.

One can take into account the sensitivity of different equipment to overcurrent by the use of circuit breakers with differing cut-off properties. They are classified with a type which describes the multiplier of the nominal current which activates the trip.

Table 3.7:

Type	Multiplier of the nominal current which activates the trip	Examples of uses
B	3 – 5	household use for lights and plug sockets
C	5 – 10	business use for lights and plug sockets
K	8 – 15	for circuits with motors or transformers
Z	2 – 3	for sensitive electrical elements
L	Old labeling system. Now split into B or C.	

Example: **Circuit breaker 16 A, B-type (3 – 5)**
a short-circuit is triggered between 48 A and 80 A

All circuit breaker have the following pictograms:
German Electro technology Association (VDE) mark of conformity
Trigger type
Nominal current
Breaking capacity (breaking capacity for short-circuit 3000 A / 6000 A or 10000 A)
Current limiting class (highest class 3 – 2 – 1)
Nominal potential

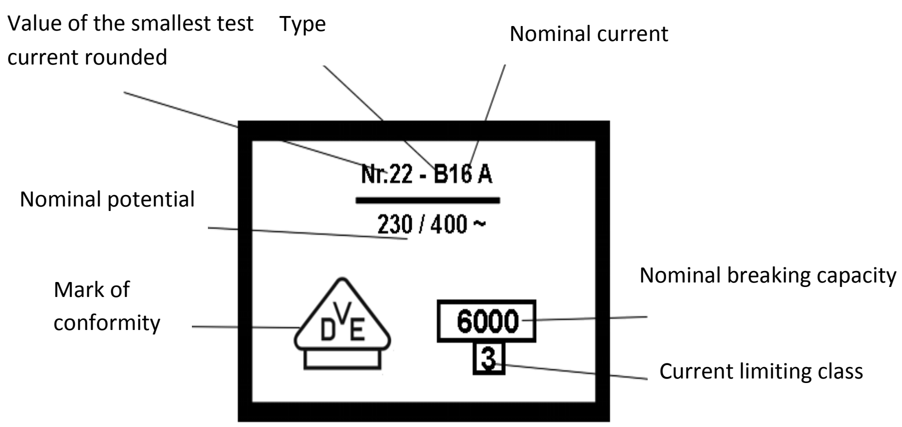


Figure 3.31: Circuit breaker rating plate

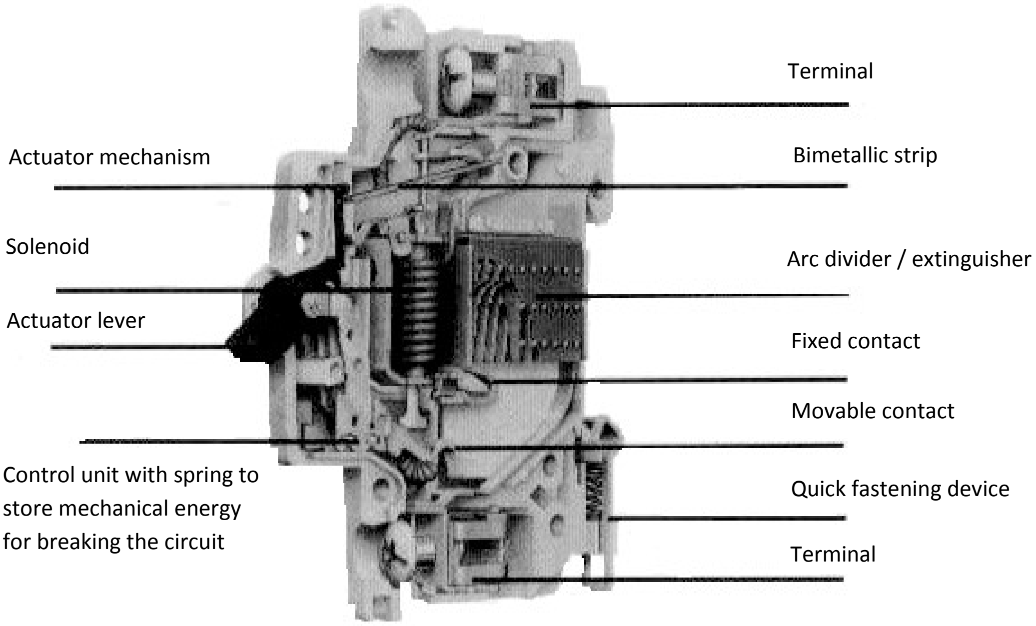


Figure 3.32: Inside a circuit breaker

3.8.2 Fuses

A fuse is another type of protection against overcurrent, and so is an alternative to the circuit breaker. It also breaks the circuit in case of an overload or short-circuit. A short-circuit needs to cut the circuit straight away, an overload only after a certain time.

Fuses are categorized according to two criteria:

1. Design:
Screw system (D and DO system)
Blade contact system (NH, HH system)

2. Class:
This is labelled with two letters:
The 1st letter indicates the range:
g = Full-range fuse (for both overload and short-circuits) - these are general purpose
a = Partial-range fuse (for short-circuits) - an associated device must provide overload protection
The 2nd letter indicates what it is designed to protected, the application category:
G(L)= wires and cables
M = switching devices
R = semiconductor devices
B = mining equipment
Tr = transformers

Fuse construction:

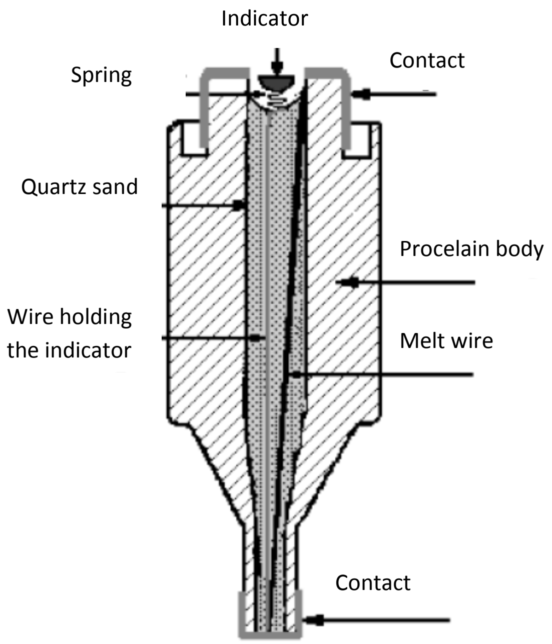


Figure 3.33: D / DO system fuse

One or more melt wires go through the quartz sand. They are attached to the contacts. The melt wire is made of silver, copper or an alloy of both metals. Alongside the melt wire, another wire, holding the indicator, leads from the bottom contact, sometimes made from constantan. The indicator is attached via a small spring. When the fuse blows, both the melt wire and the wire holding the indicator melt, the latter giving up the colored indicator.

There are 4 types in total:

1. **DIAZED-system** (also called the D-system), this is an older system, nominal potential AC and DC 500 V

Size	Nominal current in A	Thread
D II	2 – 25	E 27
D III	35 – 63	E 33
D IV	80 – 100	R 1¼ inch
D V	125 – 200	R 2 inch

Table 3.8:

2. **NEOZED-system**

a smaller, newer system (also called the DO-system) nominal potential AC 400 V, DC 250 V

Size	Nominal current in A	Thread
D 01	2 – 16	E 14
D 02	20 – 63	E 18
D 03	80 – 100	M 30*2

Table 3.9:

3. **NH-system**

low potential – high power fuses with for currents of 6 to 1250 A:

Size	Nominal current in A	Approximate blade length in mm
NH 00	6 – 100	78
NH 0	35 – 160	125
NH 1	80 – 250	135
NH 2	125 – 400	150
NH 3	315 – 630	150
NH 4	500 – 1000	200
NH 4a	500 – 1250	200

Table 3.10:

NH-fuses have blade-style terminals. One can only install or remove their melt wires with an insulated handle with attached lower arm protection. To avoid the use of incorrect melt wires, the diameter of the bottom contacts is different for different nominal currents. This means that melt wires with higher nominal currents do not fit into units with lower nominal currents.

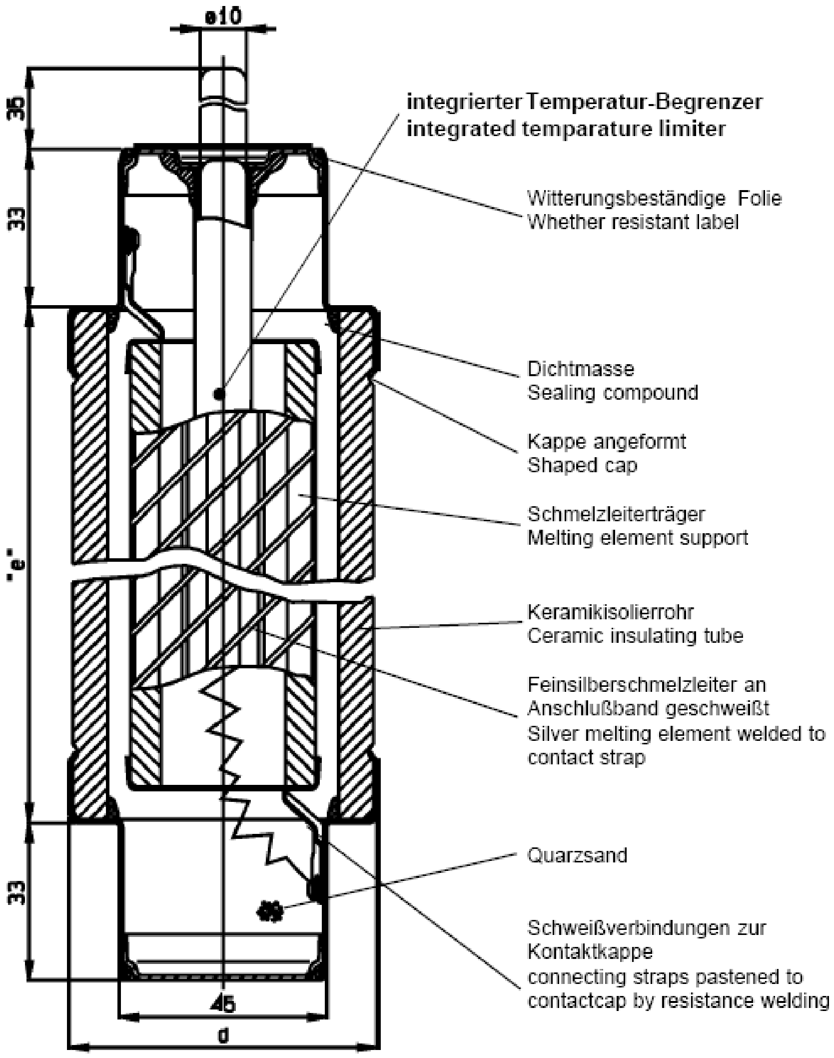
Figure 3.34: NH-fuse

4. **HH-System**

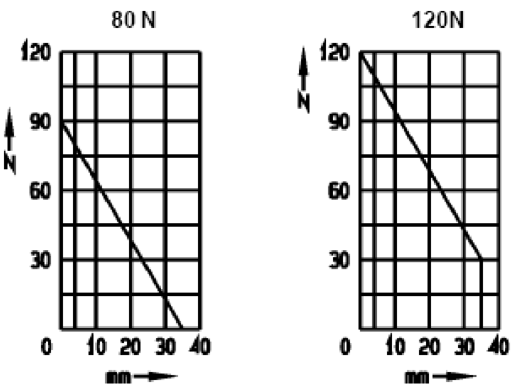
(High potential and high power fuses)

These fuses are used for short-circuit protection with nominal potentials of 3.6 kV – 36 kV. They have several silver melt wires arranged in parallel in the quartz sand. When there is an overload, the melt wire and the holding wire melt the quartz sand extinguishes the arc. The striker is released as the holding wire melts triggering the trip switch or a signaling unit. The trip switch is then immediately disconnected at all poles.

HH-Teilbereich- Sicherungseinsatz; Längsschnitt
HV-Back up fuse link; vertical cut



Kraft / Weg - Diagramm
force / distance diagram



Nennspannung rated voltage kV	"e" mm
6	192
10	292
20	442
30	537

Figure 3.35: HH-fuse

5. **Fine-wire fuses**

   There are also miniature fuses such as glass fuses or fine-wire fuses, e.g. power supply units and measuring instruments.

   We differentiate between the following trigger properties:

   FF – super fast-blow
   F – fast-blow
   M – medium
   T – slow-blow
   TT – time delay

Figure 3.36: Fine-wire fuses

3.9 Protective measures against electricity

When dealing with electricity, there is always a danger of electric shocks. Protective measures must be taken.

As soon as a person touches a live part, current will flow through his or her body. This interferes with the normal electrical actions within the body and can lead to death. Not only the **strength of the current**, but also the **exposure time** is important.

Strength of the current I in mA (t=10 s)	up to 0.5 mA	0.5 – 10 mA	10 – 50 mA	over 50 mA
Reaction	No reaction	No damaging effects (e.g. warm feeling)	Danger of ventricular fibrillation	Effects likely to be fatal

Table 3.11:

DIN/VDE 0100 demands that protective measures are taken both against **direct contact** and **indirect contact**.

Overview of protective measures according to DIN VDE 0100 Part 410

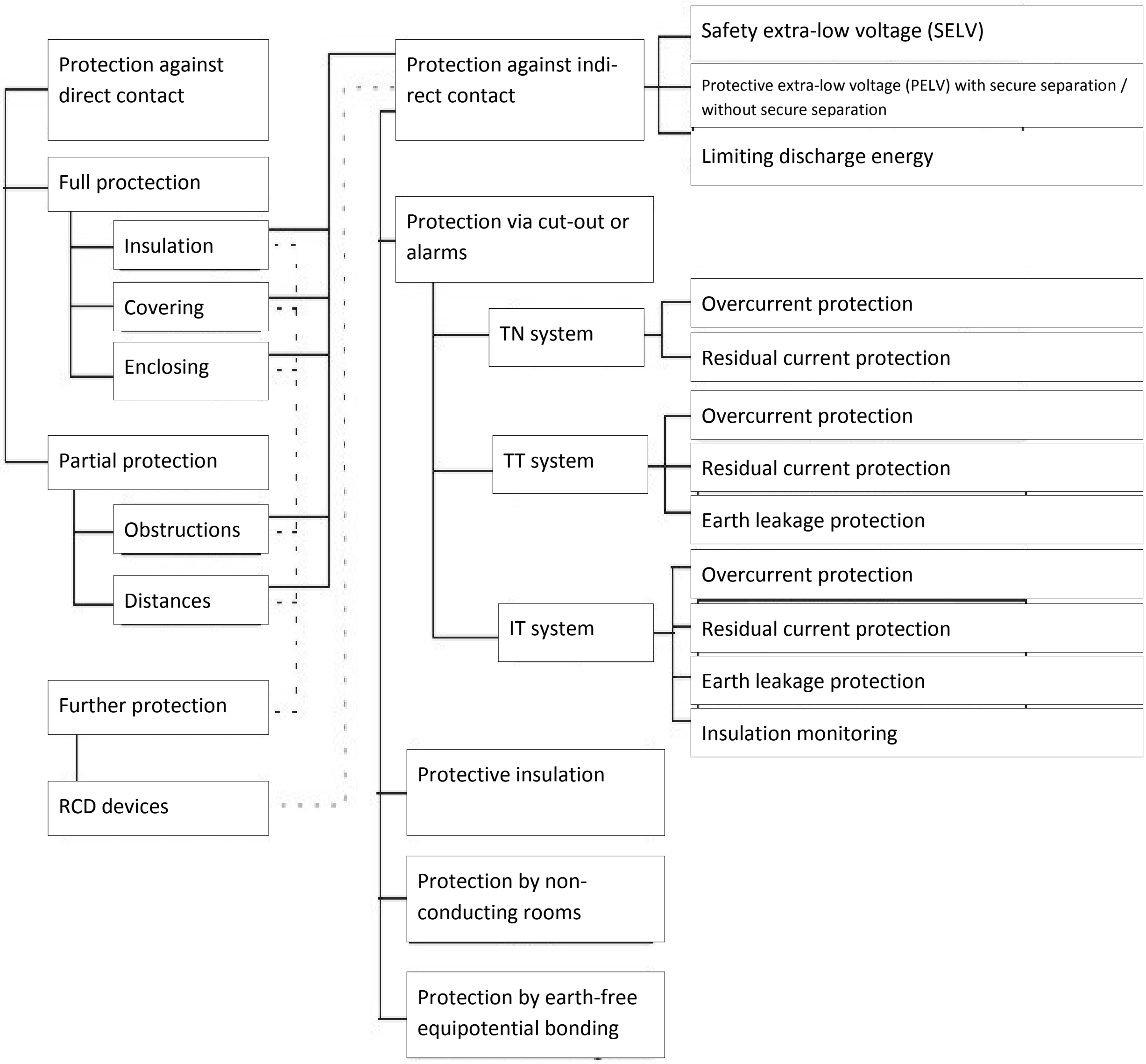


Table 3.12:

3.9.1 Protection against direct contact

This is protection from contact with live parts during the course of normal operations. It can be full or partial protection. Partial protection is only to protect against accidental contact (VDE 0100 Part 200/A.8.1).

3.9.1.1 Full protection

- **Insulating live parts**
  (protective insulation) is done by adding extra insulation on top of the basic insulation or strengthening the basic insulation. In the event of a failure of the basic insulation, no dangerous currents can flow.

- **Protection by covering or enclosing**
  must offer full protection against direct contact with live parts. This is not the case when large openings are exposed when replacing parts, e.g. lamp sockets, or when a large opening is necessary for normal operation.
  Precautions must be taken to avoid accidental contact.

3.9.1.2 Partial protection

- **Protection with obstacles or distance**
  is partial protection against direct contact.
  This is only allowed in special cases e.g. in closed electric facilities.

3.9.2 Protection against indirect contact

3.9.2.1 SELV (Safety extra low voltage)

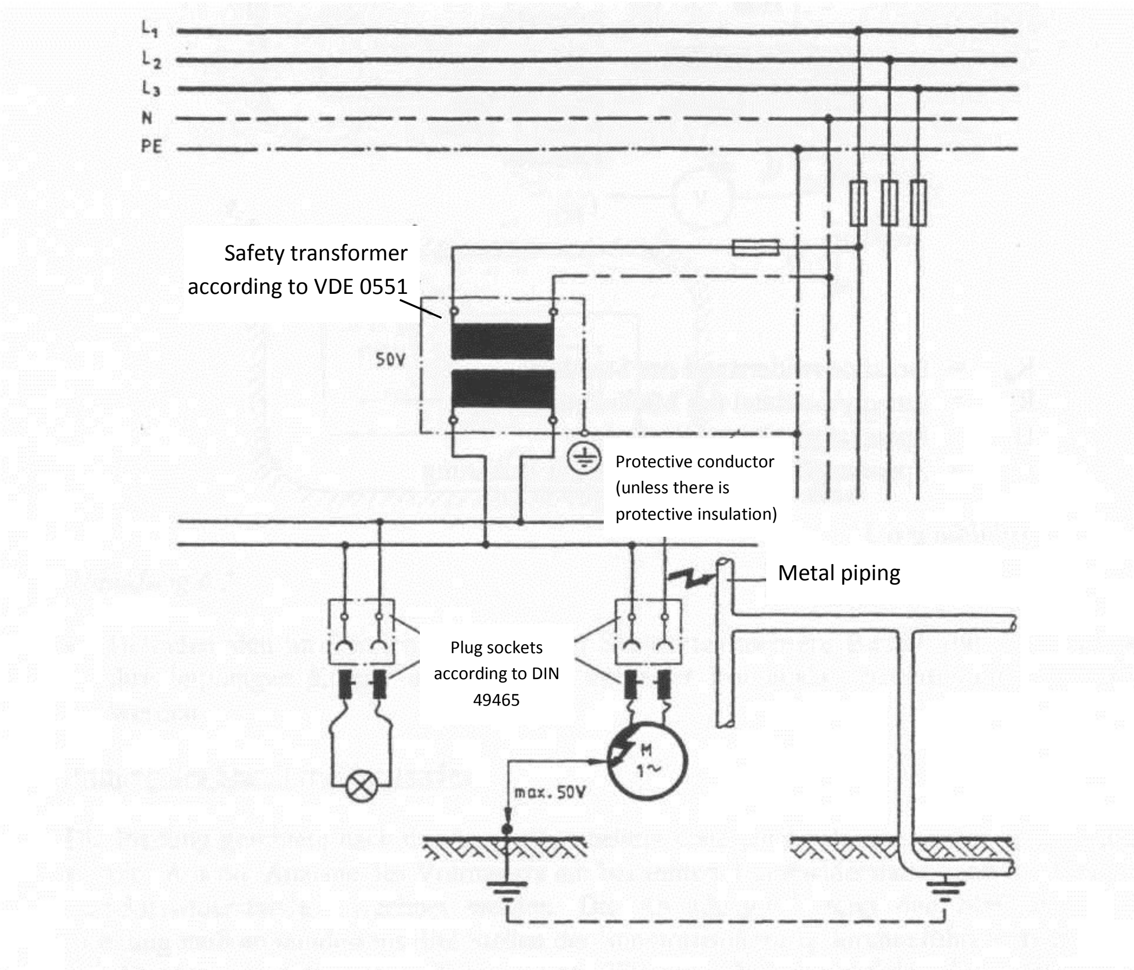


Figure 3.37: The use of SELV

SELV is using an unearthed potential of 50 V AC or 120 V DC as a protective measure. However, protection against direct contact must be ensured. Covers and enclosures must have an IP rating of at least 2 x according to DIN 40050 or insulation which with withstand a potential of 500 V AC for 1 min.

If there is no protection against direct contact, then the potential is restricted to a maximum of 25 V AC and 60 V DC. For intercoms and buzzers, the operational potential is limited to 12 V AC. Due to this low voltage, any current flowing through humans or animals will not be dangerous.

This measure protects against both direct and indirect contact.

Only power sources which guarantee that the potential will not exceed the maximum allowed can be used to generate SELV.

Also to note is:
Plugs for SELV devices must not be able to be inserted into sockets delivering other (higher) potentials.

3.9.2.2 PELV (Protected extra low voltage)

Two other forms of extra-low voltage are protected extra-low voltage (PELV) **with safe disconnection** and **functional extra low voltage (FELV) without safe disconnection.**

Where necessary, metal housings may be earthed here.

Protected extra-low voltage (PELV) with safe disconnection does not need to provide protection against contact.

Functional extra low voltage needs to include protection against indirect contact, i.e. the equipment must be covered by the upstream protection. Otherwise there are the same rules and conditions as for SELV.

3.9.2.3 Limiting discharge energy

Detailed provisions are being worked on, in which limits will be set for the discharge energy for AC and DC potentials, amongst other pulsing and irregular potentials.

At the moment, it is only set that protection against direct contact is not necessary when the discharge energy is no larger than 350 mJ or if the short-circuit current at the workplace is at most 3 mA for AC (root mean square) or 12 mA for DC.

3.9.2.4 Electrical separation

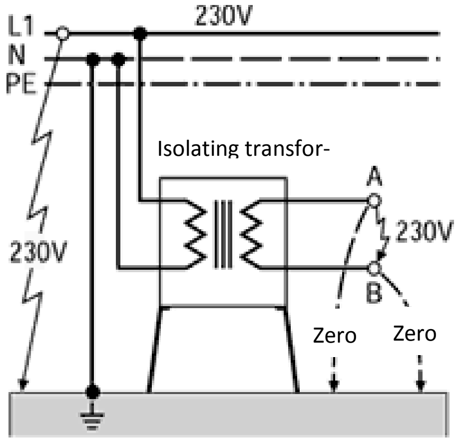


Figure 3.38: Electrical separation

Electrical separation separates the circuit of a consumer with a potential of at most 400 V AC from the electricity supply using an isolating transformer or motor generator. In this way, there is a full electric separation between the primary and secondary networks. The secondary has a floating ground. This means it has no relationship to the earth potential.

The potential between L and N lines or between L and the earth is 230 Volts.
The potential between the secondary terminals A and B is 230 Volts.
The potential between the secondary terminals A or B and the earth is zero.
The nominal potential on the secondary side must not exceed 250 V AC for two-pole power tools.
Only power tools with a nominal current of 16 A or less can be connected to an isolating transformer.
Portable isolating transformers must have protective insulation.

The secondary circuit of the isolating transformer may not be earthed. For work inside boilers, the isolating transformer should be placed outside the boiler.

3.9.2.5 Protective insulation

Protective insulation is adding insulation further than that needed for operation, to give protection against contact with live parts. Metal housings can be internally or externally fully insulated. The additional insulation must be unbroken. One can not leave a gap, even for a switch socket. Equipment with protective insulation has just one, two-pole connecting cable. They are labeled with two squares, one inside the other. Protective insulation fits with Protection Class II.

Protection classes
We will quickly describe **protection classes**, which are determined for all electrical equipment by DIN EN 61140 (DIN VDE 0140-1). They are used to categorize and label electrical equipment and are based on safety measures to avoid electric shocks.

There are **four protection classes**. There are symbols for labeling equipment with the correct class. The protection for the different classes is described in DIN EN 61140 (DIN/VDE 0140-1):2003-08, Paragraph 7.

Protection Class 0

There is no particular protection against electric shocks further than basic insulation. Protection Class 0 is not allowed in Germany. Class 0 has no symbol.

Protection Class I

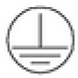


Figure 3.39: Symbol Class I

All electrically conductive parts of housings are to be connected to the permanent electrical installation's earthing system. Portable devices of Protection Class I have a plug with connection to earth. The connection to the earth must be achieved in such a way that when the plug is plugged in, the earth is connected first, and when it is removed, the earth is disconnected last. The measures protect against electric shocks when there is a short to the housing.

Protection Class II

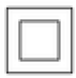


Figure 3.40: Symbol PCII

Protection class II has strengthened or double insulation and no connection to the earth. These measures are also called protective insulation. Even if they have a conductive surface, strengthened insulation protects against contact with live parts.

Portable devices of Protection Class II have 2-wire cables with a sealed plug that has no connection to earth (Europlugs or Schuko plugs without earth).

Protection Class III

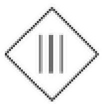


Figure 3.41: Symbol PCIII

Equipment of Protection Class III works with SELV and therefore do not need any extra protection.

3.9.2.6 Protection by non-conducting rooms

Contact with several conductive parts at the same time should be avoided. They may have differing potentials due to failure of the basic insulation.

Bodies (e.g. the housing of electric devices) are normally to be built so that simultaneous contact with two bodies or one body and a conductive part of another is not possible.

In a non conductive room there must be no earth connected to built in equipment of Protection Class 1, or to plug sockets.

The resistance of insulated floors and walls must be at least 50 kΩ for a maximum nominal potential of 500 V AC or 750 V DC and 100 kΩ for higher nominal potentials.

3.9.2.7 Protection by earth-free equipotential bonding

Dangerous contact potentials are avoided with earth-free equipotential bonding.
All bodies and other conductive parts which can be touched at the same time must be connected with an equipotential bonding conductor.

The equipotential bonding system must be neutral (without earth).
The minimum cross-section of the equipotential bonding conductor should be

- a separate, protected line of 2.5 mm^2 Cu or 4 mm^2 Al
- a separate, non-protected line of 4 mm^2 Cu and
- a shared line 0.5 mm^2 Cu for insulated power lines.

3.9.2.8 Protection via cut-out or alarms

Protection via cut-out or alarms requires a protected earth, PE. The PE conductor connects all pieces of equipment together giving them the same potential. In this way, no dangerous contact potentials can arise. In case of a short to the housing or earth fault, the current should flow through the PE conductor and trigger the protection measures. A precondition is a low resistance continuous connection to the PE conductors.
In the TN, TT and IT networks specified in DIN VDE 0100, automatic cut-off or alarms are assured. The network is protected by overcurrent protection, residual current devices or insulation monitoring. The networks are labeled with a two-letter code.

The first letter stands for the relationship of the earth and power source, and the second for the relationship of the earth and consumer.
The first letter can be either T (terra = earthed) or I (isolation).
The second letter can be N or T.

N means there is a connection, via a PE conductor, to the earth of the power source.
T means that there is a direct earth connection.
In combination, there are TN, TT and IT networks. A TN network, however, can have 4 or 5 lines.
A 4-line TN network (L1, L2, L3, PEN) is called a TN-C network,
A 5-line TN-System (L1, L2, L3, PE, N) is called a TN-S network.

TN-System

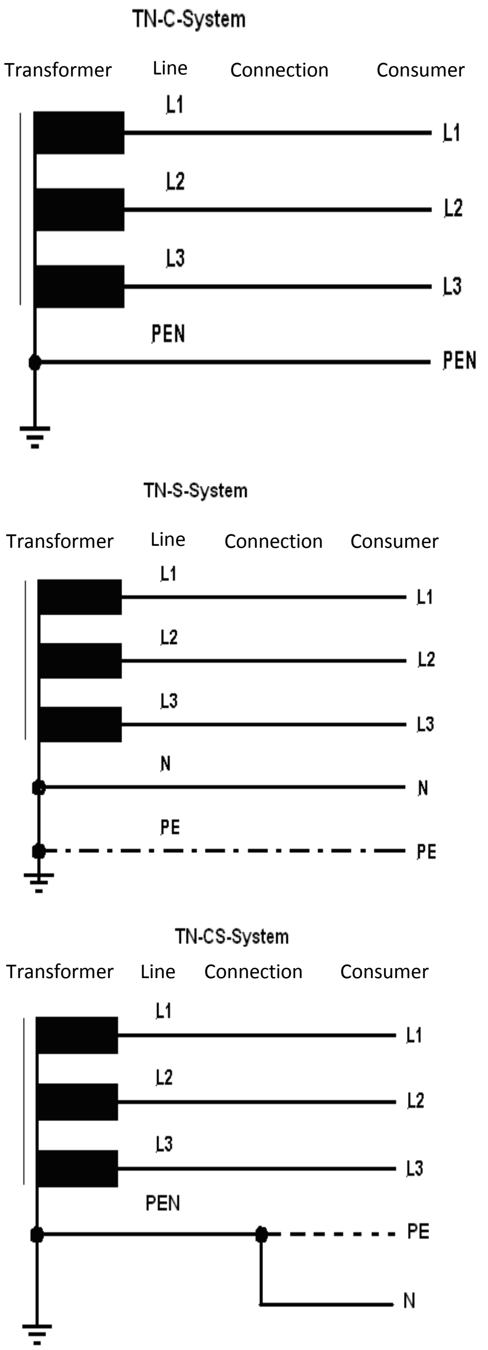


Figure 3.42: TN-C network, TN-S network, TN-CS network

TN networks are the most common form of supply network. The power source (normally a transformer) is earthed at the star point. The bodies of all equipment are connected to either a PEN or PE conductor, depending on the type of TN network. This conductor connects them to the earth of the power source.

In a TN-C network (French: Terre Neutre–Combiné), overcurrent protection (motor circuit breakers, fuses) ensures safety from indirect contact. In such a system, any short to the housing of Protection Class I equipment becomes a short-circuit. A current high enough to cut off the supply is ensured by using a low-resistance loop resistance from the line L and PEN conductor.

In a TN-S networks (French: Terre Neutre–Separé), a residual current device (RCD) protects against contact potentials being too high.

Residual current devices (RCDs)

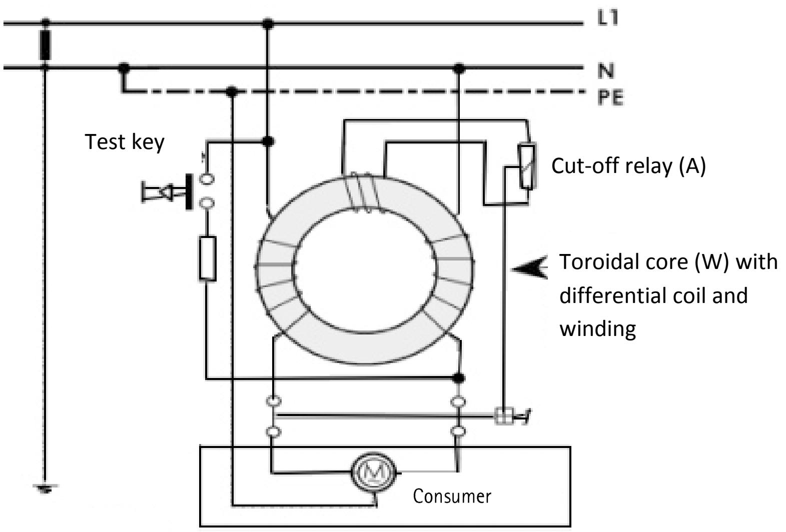


Figure 3.43: The main circuit of a RCD

RCDs disconnect electrical equipment in case of a short circuit. They are connected in the junction box alongside overcurrent protection.

At the heart of an RCD is a core-balance transformer (W). This sums up all the current flowing to and from the consumer. If a current flows from the consumer to earth, then the sum of the currents flowing to and from the consumer is not zero. This is detected in the core-balance transformer as a current difference ΔI. This triggers the RCD and cuts off the current. The core-balance transformer consists of a toroidal core wound with a crystalline or nanocrystal magnetically soft band.

RCD can be bought that trigger at ΔI = 10 mA, 30 mA, 300 mA, 500 mA or 1 A.

Older RCDs were only designed to monitor AC residual current. Modern ones are also sensitive to pulsed current and are thus able to deal with modern electrical equipment (all-current sensitive). This extra sensitivity is achieved with special magnetic materials in the toroidal core. The relevant norm is DIN VDE 0664.

On average, 230 V applied to a human body produces a current of about 80 mA which is enough to kill. This means that only RCDs with a sensitivity of 10 or 30 mA are useful for protecting people. The larger models are for fire-prevention or protection against problematic earthing.

The test key simulates a problem and so one can check that the RCD is functioning properly. This must be done at least once every 6 months and manufacturers recommend a monthly test.

TT systems

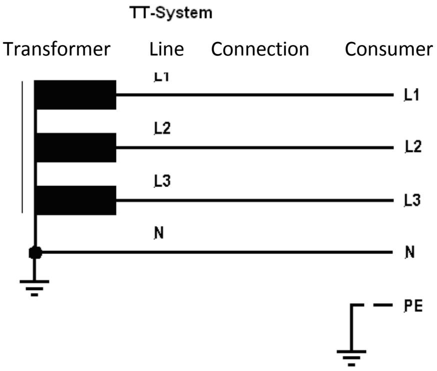


Figure 3.44: A TT system

The secondary side of a transformer is connected in a star configuration. The star point is earthed and is used as a separate neutral line (N).

Consumers must have their own earth built in.

To protect against indirect contact with overcurrent protection devices, the resistance to the earth must be very low. This takes much effort and costs. A **protective earth** can be used, but due to the problematic earthing conditions, it is limited to a current of 6A. If one wants a higher current, one needs to use an **RCD**. The trigger currents for RCDs are also affected by the earthing conditions.

The trigger current I_Δ of the RCD can be calculated with the formula: $\mathbf{I_\Delta = U/R}$. Here, U is the highest allowable contact potential (e.g. 50 V AC) and R is the earthing resistance of the electrical consumer equipment (e.g. 80 Ω). In our example, there is a difference in the current of 0.625A. This means that a normal RCD with trigger current of 0.5 A can be used.

IT systems

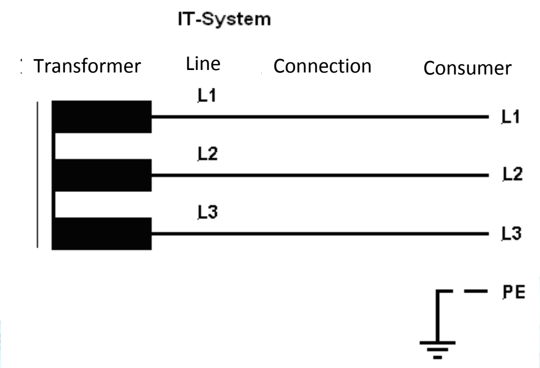


Figure 3.45: An IT system

The transformer is not earthed in an IT system. The conductive parts of the consumer's housing are earthed. When there is a fault (e.g. in the insulation) the conductor to the earth can not form a closed circuit. The user of the defect equipment cannot therefore have a dangerous current through his or her body. In order to detect the fault, the resistance of the insulation must be checked and when it is lower than a certain level, it must be optically and acoustically signaled. The acoustic alarm may cease or be stopped but the optical alarm must remain until the fault is repaired. When a fault to earth occurs, there is no disconnection, just an alarm. There is normally enough time to find and repair faults. It there is a second fault to earth, then overcurrent protection devices or RCDs do disconnect the current.

4. Stress

4.1 Tensile stress and stress limits

This section deals with the mechanics of **solid materials**. This means we consider how solid objects behave when subjected to forces. We will look at some properties of materials which have been found by testing, in particular their strengths.

Note: By *strength* we mean the resistance of a material to being deformed or destroyed.

Such a resistance is necessary when a component is subjected to external forces. If these forces are too large, then the component will be destroyed. One says then that the stress was too high. Table 4.1 shows a categorization of the forces affecting a component.

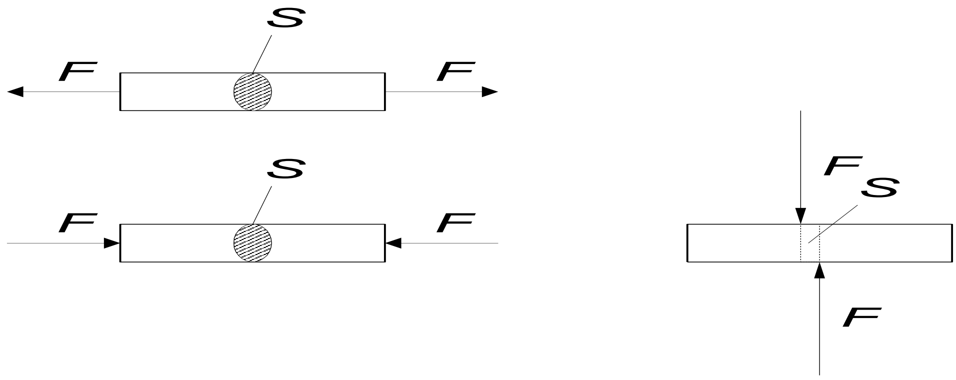
Table 4.1: Stress on solid bodies

Type of stress	Deformation	Possible destruction	Name of the stress
1. Tension	Extension	Tearing	Tensile stress σ_z
2. Compression	Contraction	Crushing	Compressive stress σ_d
3. Shearing	Shearing	Shearing off	Shear stress τ_a
4. Buckling	Buckling	Buckling	Buckling stress σ_K
5. Bending	Bending	Breaking off	Bending stress σ_b
6. Torsion	Turning	Twisting off	Torsional stress τ_t

Firstly we will consider the term **stress** more closely. Look at table 4.1 again. We can see that there are different symbols for stress, namely σ (sigma) and τ (tau). The types of stress in table 4.1 are split into two groups. In this unit, we will consider the first group: tension, pressure and shearing. We can see an important difference in table 4.2.

Table 4.2: Principal and shear stress

	Direction of force	Symbol	
Normal stress	perpendicular to the plane being stressed	σ	Figure 4.1 a)
Shear stress	parallel to the plane being stressed	τ	Figure 4.1 b)



a) Tension and compression (*F* I *S*) b) Shearing (*F* % *S*)

Figure 4.1: Normal and shear stress

Note: the	Normal stress σ is calculated from the relationship between the force on the axis, *F,* and cross-sectional area stressed, *S*.

Tensile stress $\sigma = \frac{F}{S}$ $\rightarrow [\sigma_z] = \frac{[F]}{[S]} = \frac{N}{\mathrm{mm}^2}$

Note:	The unit of *stress,* also called *mechanical stress* is Newtons per square millimeter.

Example 4.1:

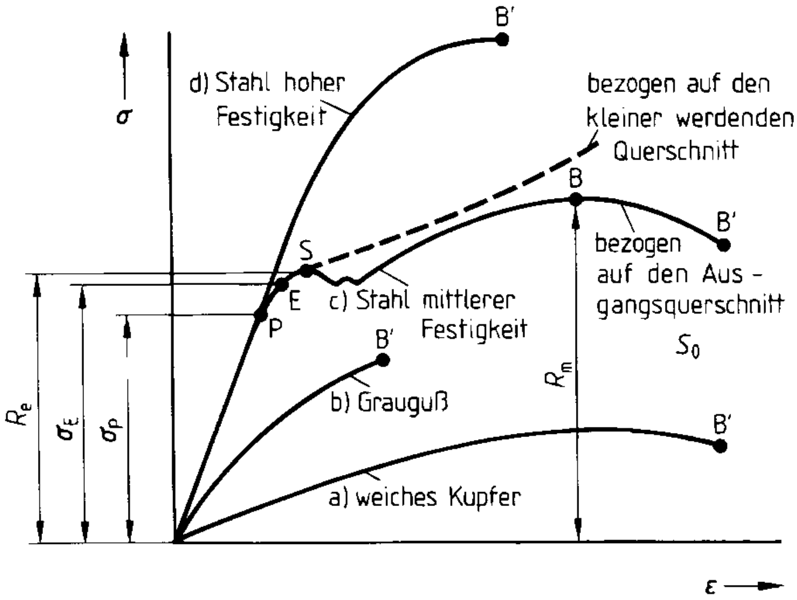
A round bar with a diameter *d* = 20 mm breaks when subjected to a stress of σ_z = 400 N/mm. What was the force *F*?

Solution:

$\sigma_z = \frac{F}{S}$ $\rightarrow$ F = $\sigma_z \bullet$ S = $\sigma_z \bullet \frac{\pi}{4} \bullet \mathrm{d}^2 = 400 \frac{N}{\mathrm{mm}^2} \bullet \frac{\pi}{4} \bullet (20\mathrm{mm})^2$ = 125,664 N

In Example 4.1, the bar is broken. This means that the **failure stress** has been reached. As this is tensile stress, it used to be given the symbol σ_{zB} *e. g.,* where the B stands for "breaking" Today, the symbol $\mathbf{R_m}$ is used.

Of course, whenever there is tensile stress, the material is stretched. We call this stretching "strain", and give it the symbol ε. Once can show the relationship between strain and stress σ on a **stress-strain curve,** the σ**,** ε **diagram**:



d) high-strength steel
c) medium-strength steel
b) cast iron
a) soft copper
taking into consideration the decreasing cross section;
based on the original cross section]
Figure 4.2: Stress-strain curves for various materials

Curve c) in figure 4.2 shows the stress-strain curve for medium-strength steel. Many other materials show a similar relationship, which is that they deform a great deal just before reaching the failure stress. One says that the material is stretching or yielding. The stress at which the material starts to yield is called the **yield point** and is given the symbol R_e. figure 4.2 shows further characteristic points which are the limits at which the behavior of the material changes. This is explained in table 4.3.
It is clear that components must not be stressed to the breaking point. If one also wants to avoid yielding, then one tries to keep the stress a good deal under the yield point. When choosing the right dimensions of components consider the following.

Note:	*stress allowed* σ_{zul} < yield point R_e < failure point R_m.

Stress limits are obtained experimentally. If these values are known, then one can calculate the stress allowed by using a **safety value.** The symbol for the safety value is (nu).

Note:	The stress permissible is calculated by dividing the stress limit, e.g. R_m or R_e by the chosen safety value υ.

Various factors are important for choosing the safety value, for example, the extent to which the component can endanger health and lives, the precision of manufacture, the composition of the materials or the precision of the determination of the load.

Note:	The safety value is chosen to be high when the consequences of a failure could be severe.

Table 4.3: Stress limits in stress-strain curves

Characteristic point	Behavior of material	Notation for the limit value
P = proportionality limit	Up to this point, the deformation is proportional to the tensile stress.	σ_p in $\frac{N}{\mathrm{mm}^2}$
E = elastic limit	Up to this point, the material deforms elastically. After the load is removed, it returns to its original length l_o	σ_E in $\frac{N}{\mathrm{mm}^2}$
S = yield point	Between Point E and this point, the material stretches when further stress is applied. When the stress is removed, there will be a permanent deformation. This means that the material no longer returns to its starting length l_o	$\mathrm{R_e}$ in $\frac{N}{\mathrm{mm}^2}$ This used to have the symbol σ_s.
B = Breaking point	From point S on, the material stretches a great deal without an increase in load. We say that the material "yields" and thus talk about the yield point. When Point B is reached, the test rod breaks.	R_m in $\frac{N}{\mathrm{mm}^2}$ This used to have the symbol σ_B = failure stress.

Stress permissible for a tough material with a pronounced yield point
(Curve c in figure 4.2)

$\sigma_{zul} = \frac{\mathrm{R_e}}{\upsilon}$ between 1.2 and 2.2

Stress permissible for a brittle material
(Curve b in figure 4.2)

$\sigma_{zul} = \frac{\mathrm{R}_m}{\upsilon}$ between 2.0 and 5.0

The considerations necessary for determining stress allowed and stress limits of materials and components can only be gone through very simply in this course. Material fatigue depends on the mechanical influences mentioned and also on:

- The type of load
- The temperature
- The duration of the stress

The fatigue limit of a material is the stress limit which can be born without loss of the nominal material properties due to fatigue. It is also dependent on whether the load is only tensile, only compressive or both together, or also contains bending and torsion. Especially critical components need to have their life-span calculated before use. Often, experiments are necessary to make a good estimate of the durability of a component.

Figure 4.3 shows a tension bar with differing cross sections S_1 and S_2

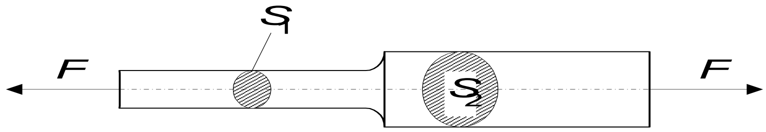


Figure 4.3: Tension bar with a differing cross section.

It is clear that the smaller cross section S_1 has a much higher stress than the larger cross section S_2 as the force *F* is spread out over a smaller area S_1. The cross section with the highest stress is called the vulnerable cross section. When making calculations for a component, it is also a case of recognizing this **vulnerable cross section.**

Note:	the highest tensile stress$_{max}$ is found at the vulnerable cross section S_{min}

Example 4.2:

The beam in figure 4.3 is made from steel. It has the following measurements R_m = 500 $\frac{N}{mm^2}$ and R_e = 260 $\frac{N}{mm^2}$. A safety value of 2.1 is to be used against yielding. In other words v_F= 2.1. The diameters are d_1= 12 mm and d_2= 20 mm. Calculate

a) The force which can be born *F*,
b) The stress in the cross section $S_2$when the force is *F*.

Solution:

a) To calculate the force which can be born, *F*, we need to calculate S_1 (vulnerable cross section). Thus

$$\sigma_{z_{zul}} = \frac{F}{S_1} \quad \rightarrow F = \sigma_{zul} \bullet S_1 = \frac{R_e}{v_F} \bullet \frac{\pi}{4} \bullet d_1^2 = \frac{260 \frac{N}{mm^2}}{2.1} \bullet \frac{\pi}{4} \bullet (12\ mm)^2 = 14{,}002.5\ N$$

b) $$\sigma_{z_{vorh}} = \frac{F_{vorh}}{S_2} = \frac{F_{vorh}}{\frac{\pi}{4} \bullet d_1^2} = \frac{4 \bullet F_{vorh}}{\pi \bullet d_1^2} = \frac{4 \bullet 14{,}002.5\ N}{\pi \bullet (20\ mm)^2} = 44.57 \frac{N}{mm^2}$$

In Example 4.2, we can see that extra indices are used. For example "pres". This is necessary for the exercise.

They mean:

prm = permissible → e.g. $\sigma_{z_{zul}}$ = tension permissible

nec = necessary → e.g. d_{erf} = diameter necessary

pres = present → e.g. F_{vorh} = force present

cho = chosen → e.g. S_{gew} = cross section chosen

Example 4.3:

Flat steel of 8 x 40 is bearing a force of *F* = 20.5 kN. It has a cross-hole (bolt hole) with *d* = 8.5 mm. Calculate the tensile stress present.

Solution:

$\sigma_{z_{vorh}} = \frac{F_{vorh}}{S_{vorh}}$ $S_{vorh} = S_1 - S_2 = 8\text{ mm} \bullet 40\text{ mm} - 8.5\text{ mm} \bullet 8\text{ mm}$

$S_{vorh} = 320\text{ mm}^2 - 68\text{ mm}^2 = 252\text{ mm}^2$

$\sigma_{z_{vorh}} = \frac{20{,}500\ N}{252\ \text{mm}^2} = 81.35\ \frac{N}{\text{mm}^2}$

4.2 Compressive stress and surface pressure

Compressive stress is normal stress, just like tensile stress, as here too the force *F* is at right angles to the area *S* (figure 4.4).

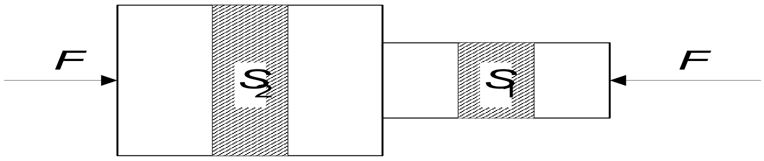


Figure 4.4: Compressive forces

In contrast to tensile forces, compressive forces work into and against each other. Otherwise, there is no formal difference in the type of load. Please note that the vulnerable cross section is used for the calculations here too. In figure 4.4, this is the cross section S_1.

Compressive stress $\sigma_d = \frac{F}{S}$ in $\frac{N}{\text{mm}^2}$

Just as for tensile stress, the stress present $\sigma_{d_{vorh}}$ must be smaller than or at most as high as the stress allowed. If one shows this in a calculation, then it is called

stress analysis $\sigma_{d_{vorh}} \leq \sigma_{d_{zul}}$

Example 4.4:

A column made from steel tubing has a load of F = 300 kN. Its external diameter is d_a = 180 mm. How high can the internal diameter, d_i be chosen to fit with a stress allowed of $\sigma_{d_{zul}} = 40 \frac{N}{mm^2}$? Create a stress analysis for the chosen diameter.

Solution:

$$\sigma_{d_{zul}} = \frac{F}{S} = \frac{F}{\frac{\pi}{4}(d_a^2 - d_i^2)} = \frac{4 \cdot F}{\pi(d_a^2 - d_i^2)}$$

This gives:

$$d_{i_{erf}} = \sqrt{d_a^2 - \frac{4 \cdot F}{\pi \cdot \sigma_{d_{zul}}}} = \sqrt{(180\mathrm{m})^2 - \frac{4 \cdot 300{,}000\,N}{\pi \cdot 40 \frac{N}{mm^2}}} = 151.16 \text{ mm}$$

When calculating dimensions, the measurements are often rounded to the millimeter. The measurements must be rounded down, as otherwise the stress allowed will be exceeded.

$d_{i_{gew}}$ = 150 mm

Stress analysis

$$\sigma_{d_{vorh}} = \frac{F_{vorh}}{S_{vorh}} = \frac{4 \cdot F_{vorh}}{\pi(d_a^2 - d_i^2)} = \frac{4 \cdot 300{,}000\,N}{\pi \cdot [(180\mathrm{mm})^2 - (180\mathrm{mm})^2]}$$

$$\sigma_{d_{vorh}} = 38.58 \frac{N}{mm^2} < \sigma_{d_{zul}}$$

Compressive stress can only occur in solid objects, for example in the column depicted in figure 4.5.

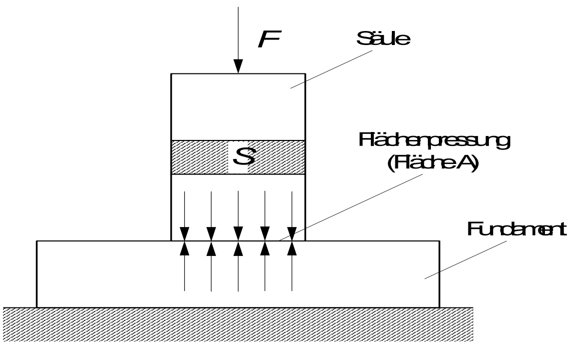


Figure 4.5: Surface pressure between two solid bodies

In such a case, there is, according to the law of action and reaction, a reaction force generated between the column and the other material (e.g. the foundation). This force is also as big as the load *F*. This there is compressive stress on the touching surfaces *A*.

Note:	The compressive stress on the contact surface between two components is called the surface pressure σ_p.

Note:	According to DIN 1304 the symbol used for the **cross section** is ***S*,** and for the area ***A*.**

Calculating the surface pressure is done from the definition – analogously to compressive stress. Thus we have the

surface pressure $\sigma_p = \frac{F}{A}$ in $\frac{N}{\text{mm}^2}$

As the parts pressing against one another are generally made of very different materials, e.g. copper and steel, the dimensions of the areas must be calculated using the smallest **allowed surface pressure** $\sigma_{p_{zul}}$ The values are determined experimentally.

Example 4.5:

In the arrangement in figure 4.5, there is a force between the two components of *F* = 150 kN. The surface pressure allowed for the column $\sigma_{p_{zul}}$s = 100 $\frac{N}{\text{mm}^2}$ and for the foundations it is $\sigma_{p_{zul}}$= 120 $\frac{N}{\text{mm}^2}$. The rectangular area is *l* = 75 mm long. Calculate the width *b* of the rectangle which is necessary.

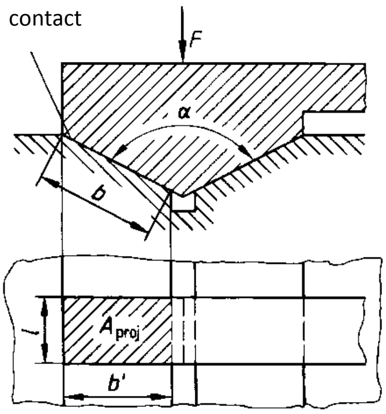
Solution:

$$\sigma_{p_{zul\,min}} = \frac{\text{F}_{vorh}}{\text{A}_{erf}} \rightarrow \text{A}_{erf} = \frac{\text{F}_{vorh}}{\sigma_{p_{zul\,min}}} = \frac{150{,}000\,N}{100\,\frac{N}{\text{mm}^2}} = 1{,}500\ \text{mm}^2$$

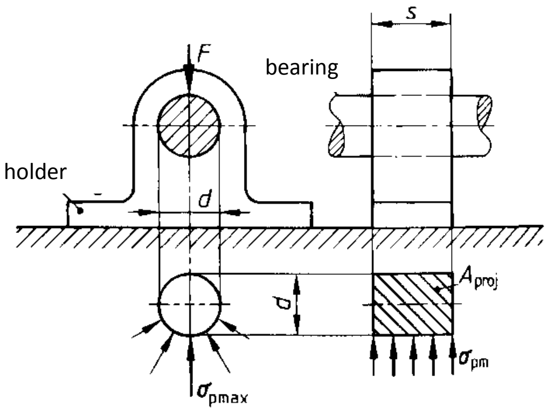
$$\text{b}_{erf} = \frac{\text{A}_{erf}}{\text{l}_{vorh}} = \frac{1{,}500\ \text{mm}^2}{75\ mm} = 20\ \text{mm}$$

In practise, it is common to find **slanting areas** being pressed together. This is the case, for example in inverted Vee guides (figure 4.6 a). Pressure due to **rounded areas** is also not uncommon, see the vertical bearing in figure 4.6 b.

Figure 4.6:



a) Pressure from a slanting area



b) Pressure from a rounded area

In such cases, the following principle holds:

Note:	Calculate the surface pressure for *slanted or curved areas* by dividing the force *F* by the *perpendicular projection* of the area A_{proj}.

As the pressures varies for curved areas, like in figure 4.6 b, one talks about the **average pressure** σ_{pm}. Figure 4.6 b also shows us that:

Note:	For cylindrical areas, the projection can be calculated as the product of the diameter *d* and the length *s*.

Example 4.6:

The shaft in the vertical bearing in figure 4.6 b has a diameter of *d* = 50 mm and the length of the area is *s* = 30 mm. The axial bearing load is *F* = 50 kN. Calculate the average surface pressure σ_{pm}.

Solution:

$$\sigma_{pm} = \frac{F}{A_{proj}} = \frac{F}{d \cdot s} = \frac{50{,}000\,N}{50\,mm \cdot 30\,mm} = 33.33\,\frac{N}{mm^2}$$

4.3 Stress and shearing

This section deals with calculations for components that are under shearing stresses. Firstly, however, we will discuss the different **loading cases** and the permissible stresses which result from them.

Earlier, we assumed that we had a constant load, that is, it did not change over time. That situation is called a **static load**. We can also call it **Load Case I**. You will already know that many components have to bear non constant loads, for example a piston rod in a motor or drive shaft. This is called a **dynamic load** and we split this into two cases: **Load Case II** and **Load Case III**. Dynamic loads are not born so easily as static loads. This means that less stress is permissible for dynamic loads. Now we will define the three load cases and then you will find the permissible stresses for two important materials depending on the load case.

Load Case I → the load is at rest, and so the stress is constant
Load Case II → the stress goes from zero to its highest value **(cyclic load)**
Load Case III → the stress changes from its highest positive value to its lowest negative value, for example between tension and compression **(alternating load)**

Table 4.4: Permissible stress for the materials St 50-2 (steel) and GC-26 (grey iron grade 26)

permissible stress in N/mm²		St 50-2	GG-26
Tension $\sigma_{z_{zul}}$ for Load Case	I	130 – 210	60 – 90
	II	85 – 135	50 – 70
	III	60 – 95	30 – 50
Compression $\sigma_{d_{zul}}$ for Load Case	I	130 – 210	150 – 210
	II	85 – 135	100 – 135
	III	60 – 95	30 – 50
Shearing $\tau_{a_{zul}}$ for Load Case	I	110 – 165	70 – 100
	II	70 – 100	50 – 75
	III	50 – 75	30 – 50

If **shear stress** is too large, then part of a component can be sheared off. The limit being τ_{aB}.

Note: Shear stress τ_a is the quotient of the external shear force *F* by the area stressed *S*.

Shear stress $\tau_a = \frac{F}{S}$ in $\frac{N}{\text{mm}^2}$

Figure 4.7 shows a practical case of a shear load, namely a pin joint.

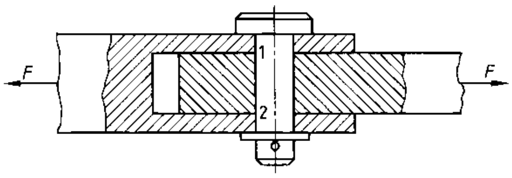


Figure 4.7: A pin under shear stress

If the pin in figure 4.7 were to fail, it would break in two places, marked 1 and 2 on the figure. We use the term double shear. We must take account of this when making calculations as the shear area is double what it would be for a single shear. The **number of shearing areas** is given the symbol *n*.

Example 4.7:

Calculate the diameter of a bolt in the fork joint depicted in figure 4.7 if the force is F = 50 kN and the permissible shear stress is $\tau_{a_{zul}} = 80 \frac{N}{\text{mm}^2}$

Solution:

$$\tau_{a_{zul}} = \frac{F}{n \bullet S} = \frac{F}{n \bullet \frac{\pi}{4} \bullet d^2} = \frac{4 \bullet F}{n \bullet \pi \bullet d^2}$$

$$d_{erf} = \sqrt{\frac{4 \bullet F}{n \bullet \pi \bullet \tau_{a_{zul}}}} = \sqrt{\frac{4 \bullet F}{2 \bullet \pi \bullet 80 \frac{N}{\text{mm}^2}}} = 19.95 \text{ mm} = 20 \text{ mm}$$

Note:	The *shear cross section* is the cross section which would be broken in a failure.

Figure 4.8 shows another example from punching technology:

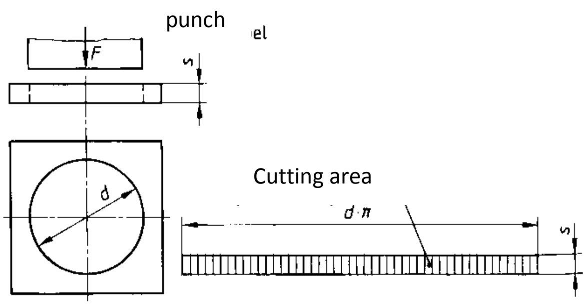


Figure 4.8: Shear areas in punching

We can see from figure 4.8 that the shear cross section is the curved surface of a cylinder. We can see this as a rolled up rectangle and calculate its area $S = d \bullet \pi \bullet s$.

Example 4.8:

In figure 4.8, a board with a diameter of d = 45 mm is being cut out of a brass sheet which has a thickness of s = 4 mm. The sheet's shear strength is $\tau_{aB} = 340 \frac{N}{\text{mm}^2}$. What **force** F_s must be generated?

Solution:

$$\tau_{aB} = \frac{F_{erf}}{S_{vorh}} \qquad \rightarrow F_s = \tau_{aB} \bullet S = \tau_{aB} \bullet d \bullet \pi \bullet s = 340 \frac{N}{\text{mm}^2} \bullet 45 \text{ mm } \pi \bullet 4 \text{ mm}$$

F_s = 192,265 N ≈ 192 kN

Note:	In practice, the presses generate a force of approximately 200 kN or more.

Example 4.9:

Angles are cut with special scissors with a section of 70 x 7. They are made of steel and cut at forging temperatures. At these temperatures, the failure point is at τ_{aB} = 200 $\frac{N}{\mathrm{mm}^2}$. For the profile 70 x 7, the cross-sectional area is quoted as S = 9.4 cm^2. Calculate the force F_s necessary, assuming that the whole area is cut at the same time.

Solution:

$\tau_{aB} = \frac{F}{S}$ $\rightarrow \tau_{aB} \bullet \mathrm{S} = 200 \frac{N}{\mathrm{mm}^2} \bullet 940\ \mathrm{mm}^2 = 188{,}000\ \mathrm{N}$

5. THE EFFECTS OF HEAT AND TEMPERATURE

5.1 Thermal expansion of solids and fluids

Heat is a form of energy with the unit Joule. Furthermore, there are the two most important temperature scales: Celsius and Kelvin. In this unit, we will look at the effects that heat and temperature have on materials and manufacturing processes. As you know from chemistry, materials consist of **elementary particles,** the atoms or molecules. These are arranged regularly in **crystalline materials** such as metals, and irregularly in **amorphous materials**. At all temperatures above absolute zero, these elementary particles move. The amount they move depends on the amount of heat.

Note:	An increase in heat energy increases the kinetic energy of the elementary particles and a decrease in heat energy decreases the kinetic energy.

We can see from this, that the elementary particles need more or less space depending on how much they are moving. Thus most materials expand when heated and contract when heat is taken away. Water is an exception to this rule, over a certain temperature range, having its highest density at 4°C. This special behavior may be called the **anomaly of water**. Figure 5.1 symbolizes the amplitude of an elementary particles oscillations:

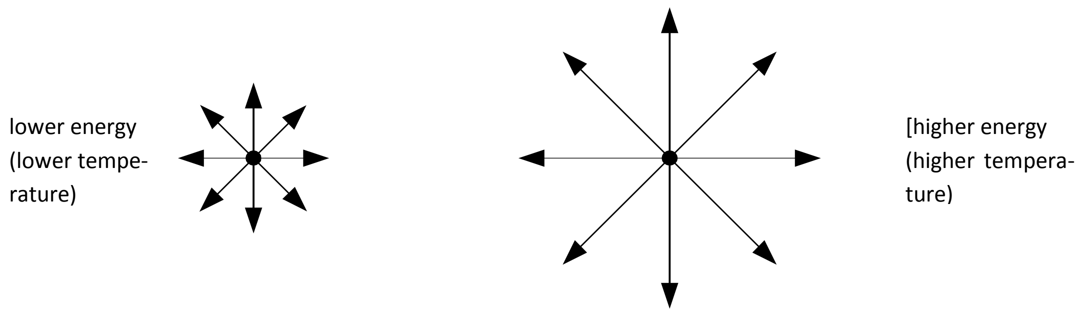


Figure 5.1: The thermal agitation of an elementary particle

The difference in the space needed which depends on heat can be seen in figure 5.1. This allows us to understand the **gas laws of Boyle-Mariotte and Gay-Lussac.** You have already learnt the following for **gases**:

Note:	When allowed to expand freely, the volume of a gas increases very fast as temperature increases (Boyle-Mariotte, Gay-Lussac, combined gas law).

Up until now, we have not explained the difference between temperature and heat. The difference is clearly shown in the experiment depicted in figure 5.2:

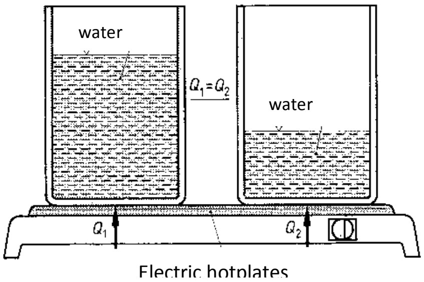


Figure 5.2: Amounts of heat

Suppose we fill two similar containers with different amounts of a liquid (e.g. water), we add the **same amounts of heat** Q_1 and Q_2 in the same time, then the temperatures will change with differing speeds. We have the same result when we use the same quantities but of different substances (e.g. water and oil). You can see different temperatures after a certain time – although the same heat energy has flowed in. Thus, in contrast to how the words are used in non-technical language, we must differentiate between temperature and heat.

Note:	Temperature is a state variable, heat is a measure of energy (thermal energy).

As we have said: The principles of the **thermal expansion** for gasses have already been dealt with. Now we wish to deal with the rules and laws of thermal expansion for liquids and solids. These can also be explained by considering the amplitude of the particles. One differentiates between **Thermal linear expansion** and **Thermal volumetric expansion**.

This difference is purely a practical one, as there are components which expand predominantly in one direction only. Examples of this would be railway tracks or pipes. For such components, only the change in length is of interest. It is different when we consider, for example, containers or compact parts of equipment. Here, we are generally interested in the volumetric expansion.

In contrast to gases, each liquid and solid has its own potential for expansion, i.e. different materials expand at different rates when the temperature changes. For **linear expansion of solids**, the important figure is the **thermal coefficient of linear expansion**. In practice, this is also called the **coefficient of thermal expansion**.

Note: The coefficient of thermal expansion α is a constant of the material and gives the linear extension undergone by a 1 m long rod when its temperature is raised by 1 degree Celsius or Kelvin.

This means that the **units of coefficients of thermal expansion** are $[\alpha] = \frac{m}{m \bullet °C} = \frac{m}{m \bullet K} = \frac{l}{K}$

The values are α sometimes very temperature-dependent. For this reason, the normal value quoted is that at room temperature, 20°C. Table 5.1 shows you some values. Further values can be found in technical handbooks or product descriptions from manufacturers.

Table 5.1: Thermal coefficients of linear expansion

Material	α **in** $\frac{m}{m \bullet K}$	**Material**	α **in** $\frac{m}{m \bullet K}$	**Material**	α **in** $\frac{m}{m \bullet K}$
Aluminium	0.000024	Glass	0.000009	Brass	0.000018
Antimony	0.000011	Gold	0.000014	Nickel	0.000013
Concrete	0.000012	Grey iron	0.000011	Platinum	0.000009
Lead	0.000029	Carbide metal	0.000005	Mercury	0.0000606
Bronze	0.000018	Copper	0.000017	Silver	0.000020
Pure iron	0.000017	Magnesium	0.000026	Steel	0.000012

From the definition of the coefficient of thermal expansion, we have that

thermal expansion $\Delta l = l_1 \bullet \alpha \bullet \Delta\vartheta$

l_1 = original length
$\Delta\vartheta$ = difference in temperature

To calculate the final length l_2 when the temperature is increased, add the thermal expansion Δl to the initial length . When the temperature decreases, the thermal expansion Δl is subtracted from the initial length l_1. Thus:

Final length $l_2 = l_1 \pm \Delta l = l_1 \pm l_1 \bullet \alpha \bullet \Delta\vartheta$

+ for an increase in temperature
– for a decrease in temperature

Example 5.1:

A steel pipe [α = 0.000012 $\frac{m}{m \bullet K}$] has a length of l_1= 25 m when it is at 10 °C. What is its length l_2

a) ϑ_2 = 40 C,
b) ϑ_2= –5 C?

Solution:

a) $l_2 = l_1 + l_1 \bullet \alpha \bullet \Delta\vartheta$ = 25 m + 25 m • (0.000012 $\frac{m}{m \bullet K}$) • 30 K = 25 m + 0.009 m

l_2 = 25.009 m (extension of 9 mm)

b) $l_2 = l_1 - l_1 \bullet \alpha \bullet \Delta\vartheta$ = 25 m - 25 m • (0.000012 $\frac{m}{m \bullet K}$) • 15 K = 25 m + 0.0045 m

l_2 = 24.9955 m (contraction of 4.5 mm)

Figure 5.3 shows the volumetric expansion of a cube for an expansion from ϑ_1 to ϑ_2. Notice that each edge expands by Δl:

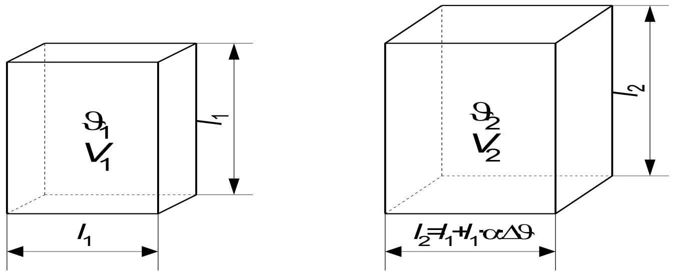


Figure 5.3: Thermal volumetric expansion

From the volume V_1 at temperature ϑ_1 the volume has grown to V_2 at temperature ϑ_2. As each edge extends from l_1 to l_2 regardless of whether it is hollow or solid, we get the following, important rule:

Note:	Hollow bodies expand to the same degree as solid bodies when the temperature changes.

If we calculate the expansion in figure 5.3 we have $V_2 = l_2^3 = (l_1 + l_1 \bullet \alpha \bullet \Delta\vartheta)^3$
which, is about

$V_2 = V_1 + V_1 \bullet 3\alpha \bullet \Delta\vartheta$

According to DIN 1304, the value 3α is called the

volumetric expansion coefficient $\gamma = 3 \bullet \alpha$ in $\frac{m^3}{m^3 \bullet K} = \frac{l}{K}$

It is important here too that a rise in temperature gives rise to expansion and a fall to contraction. Then we have that the

Final volume $V_2 = V_1 \pm V_1 \bullet \gamma \bullet \Delta\vartheta$ + for an increase in temperature
– for a decrease in temperature

This equation is generally valid, i.e. it does not depend on the shape of the body. It is also valid for very irregular shapes.

Example 5.2:

A steel ball [α = 0.000012 $\frac{m}{m \bullet K}$] has a diameter of d_1 = 10 cm. It is heated by $\Delta\vartheta$ = 300°C. What is its final volume V_2?

Solution:

V_2 = V_1 + $V_1 \bullet \gamma \bullet \Delta\vartheta$; $V_1 = \frac{\pi}{6} \bullet d^3 = \frac{\pi}{6} \bullet (10\ cm)^3$ = 523.599 cm^3 ; $\gamma \bullet 3$

V_2 = 523.599 cm^3 + 523.599 cm^3 • 3 • 0.000012 $\frac{m}{m \bullet K}$ • 300 K = 529.254 cm^3

Thermal expansion also plays an important role for liquids. Table 5.2 shows some volumetric expansion coefficients:

Table 5.2: Volumetric expansion coefficients

Material	γ **in** $\frac{m^3}{m^3 \bullet K}$	**Material**	γ **in** $\frac{m^3}{m^3 \bullet K}$	**Material**	γ **in** $\frac{m^3}{m^3 \bullet K}$
Alcohol	0.0011	Mercury	0.000182	Oil of turpentine	0.0097
Benzine	0.0014	Nitric acid	0.00124	Toluene	0.00108
Glycerine	0.0005	Hydrochloric acid	0.00030	Water	0.00018
Machine oil	0.00076	Sulphuric acid	0.00056		

Example 5.3:

1,000 litres of benzine in a barrel are warmed 20 C by the sun to 65 C. How many more litres volume has the benzine after being warmed and what is the technical rule one can take from this?

Solution:

V_2 = V_1 + $V_1 \bullet \gamma \bullet \Delta\vartheta$ = 1,000 l + 1,000 l • 0.0014 $\frac{m^3}{m^3 \bullet K}$ • 45 K

V_2 = 1,000 l + 63 l = 1.063 l

Δl = 63 l

The volume is not 63 litres more.

Note:	*Rule:* Containers for liquids must never be completely filled or the container must be designed so that the liquid has room to expand e.g. a riser.

If one does not heed this rule, then heating can lead to the destruction of the container or at least a significant deformation. The term used here is thermal stress. Figure 5.4 shows a further structure in which **thermal stress** can appear: a pipe.

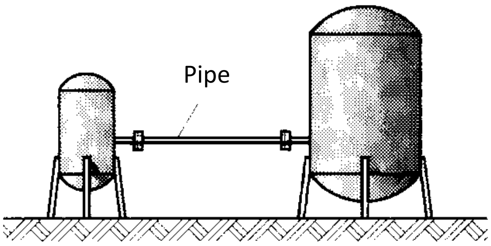


Figure 5.4: A pipe between two containers

This **thermal stress** can only be avoided by giving the liquid the possibility to expand. In figure 5.4, this could be done with an expansion joint in the pipe or the possibility of sliding one of the containers, for example on rollers.

Thermal expansion also plays a role for measuring, for example with a slide gauge or micrometer. If precision is necessary for the measurements, then they are taken in special rooms. The temperature can be kept constant in these rooms. The **measuring temperature** is normally 20 C and it is called the **technical normal temperature**.

Note:	For *fine measurements* use a reference temperature (normally 20 C). In this way, the thermal expansion of the measuring equipment and work pieces are reconstructible factors.

A further important use of thermal expansion of contraction is found in casting. The **degree of shrinkage** indicates by what percentage the measurements of castings will shrink when they solidify and cool to room temperature.

Example 5.4:

Grey iron has a linear degree of shrinkage or 1%. If we want to make a piece of length l_2 = 742, how long should the original cast be?

Solution:

$l_2 = l_1 - 0.01 \cdot l_1 = 0.99 \cdot l_1 \rightarrow l_1 = \frac{l_2}{0.99} = \frac{742\ mm}{0.99}$

$l_1 \approx 749.5$ mm

5.2 Thermal conduction

At the end of this section, we look quickly at the large area of **thermal conduction**. Figure 5.5 makes an important law of nature clear:

Note:	Without extra expenditure of energy, heat can only flow from a body of higher temperature to one of lower temperature.

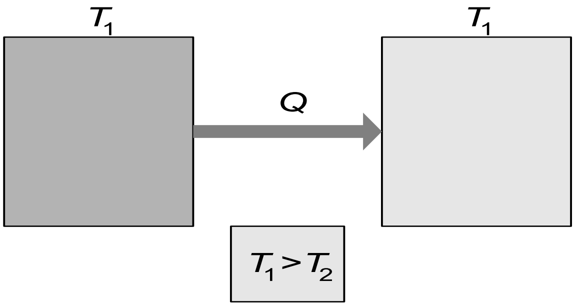


Figure 5.5: Thermal conduction in the direction of lower temperature.

In practice, we consider

good thermal conduction	→ for heating and cooling processes, e.g. for a flow of heat from a radiator into a room;
poor thermal conduction	→ for example for heat accumulators or piping with high or low temperatures. Poop thermal conduction is achieved with **thermal insulation** i.e. protection against the loss of heat.

Example: Insulation of a house wall with foam polystyrene.

We differentiate between

Insulation for warmth	→ the object temperature is higher than the ambient temperature. Example: A family home
Insulation for cold	→ object temperature is lower than the ambient temperature. Example: Cold room.

There are many comprehensive regulations concerning thermal insulation, for example the **Heat Insulation Ordinance** and VDI Guideline 2055: "Insulation for warmth and cold in commercial and domestic equipment".

Note:	Heat loss can be limited with thermal insulation but not stopped completely. The insulation provided by a thermal insulator depends on the materials used and its thickness.

We consider two essentially different mechanisms for transferring heat: **conduction** and **radiation**.

Thermal conduction: The heat is transferred directly between neighbouring parts for example, molecules, solid bodies, liquids, gases or vapours via direct contact. For the transfer of heat via conduction, note that there are good conductors (all the metals) and poor conductors e.g. insulating materials. This is similar to electrical conduction.

Thermal radiation: The energy is transferred from a place of higher temperature to one of lower temperature in small indivisible units, the **energy quanta**. This requires no material to pass through – this is how the Sun's radiation comes through empty space to get to the Earth.

Finally, note that there is a great deal of literature which deals with calculating thermal conduction or designing insulation, some of which is given at the end of this booklet.