Contents

QUANTITIES, THEIR SYMBOLS AND UNITS

Quantity	Symbol	Unit
Absolute pressure	p_{abs}	N/m^2 = Pa, bar
Absolute temperature	T	K
Work	W	Nm, J
Atmospheric pressure	p_{amb}	N/m^2 = Pa, bar
Acceleration	a	m/s^2
Work done for acceleration	W_a	Nm
Kinetic energy	W_{kin}	Nm
Bending stress	σ_b	N/mm^2
Shear strength	τ_{aB}	N/mm^2
Temperature in Celsius	ϑ	°C
Strain	ε	1
Density	ρ	kg/m^3
Work for turning	W	Nm
Turning power	P	W
Torque	M, M_d	Nm
Frequency of rotation	n	Min^1
Pressure	p	N/m^2 = Pa, bar
Pressure energy	W	Nm, J
Compressive stress	σ_d	N/mm^2
Diameter	d	m, mm
Electrical work	W	Ws, kWh
Electrical energy	W	Ws, kWh
Electrical charge	Q	As
Electrical power	P	W
Electrical potential	U	V
Electrical current	I	A
Electrical resistance	R	Ω
Energy	W, Q, E	Nm, J, Ws
Acceleration due to gravity	g	m/s^2
Surface pressure	σ_p	N/mm^2
Speed	v	m/s
Weight	F_G	N
Dynamic frictional force	F_R	N
Coefficient of dynamic friction	μ	1
Static frictional force	F_{R0}	N
Coefficient of static friction	μ_0	1
Hydraulic power	P	W
Hydrostatic pressure	p	N/m^2 = Pa, bar
Kinetic energy	W_{kin}	Nm
Buckling stress	σ_K	N/mm^2
Force	F	N
Diameter	d	m, mm
Circumference	l_u	m, mm
Length	l	m
Coefficient of linear expansion	α	m/(m • K) = 1/K
Power	P	W, kW
Mass	m	kg
Mechanical work	W	Nm, J

Mechanical energy	W	Nm, J
Mechanical power	P	W
Mechanical efficiency	η	1, %
Normal force	F_N	N
Standard acceleration due to gravity	g_n	m/s^2
Potential energy	W_{pot}	Nm
Cross section	S, A	mm^2
Radius	r	m, mm
Frictional force	F_{R0}, F_R	N
Coefficient of friction	$\mu_0,\ \mu$	1
Resultant force	F_r	N
Shear stress	τ_t	N/mm^2
Cutting speed	v_c	m/s, m/min
Safety factor	v	1
Specific resistance	ρ	$\Omega \bullet mm^2/m$
Yield point	R_e	N/mm^2
Flow velocity	w	m/s
Difference in temperature	$\Delta\vartheta, \Delta T$	°C, °K
Thermodynamic temperature	T	K
Torsional stress	τ_t	N/mm^2
Overpressure	p_e	N/m^2 = Pa, bar
Frequency of rotation	n	s^{-1}
Circumferential speed	v_u	m/s
Volume	V	m^3
Volumetric expansion coefficient	γ	$m^3/(m^3 \cdot K) = 1/K$
Flow rate	$V°$	m^3/s
Coefficient of thermal expansion	α	$m/(m \cdot K) = 1/K$
Heat = heat energy	Q	J, kJ
Distance	s	m
Angular velocity	ω	$rad/s = s^{-1}$
Efficiency	η	1, %
Time	t	s, min, h
Tensile strength	R_m	N/mm^2
Tensile stress	σ_z	N/mm^2

Arithmetic

This includes number theory and the use of letters to represent variables.
This includes arithmetic operations like:

- Addition
- Subtraction
- Multiplication
- Division

Exponentiation as well as taking roots.

Algebra

This is the theory of equations (especially with letters standing for variables).

Geometry

This deals with the calculation of areas and volumes of bodies with various shapes and trigonometry (triangulation).

1 IMPORTANT BASIC KNOWLEDGE OF MATHEMATICS

1.1 Basic arithmetic

Calculation type	Arithmetic operation	Arithmetic operator	Arithmetic group
Addition **Subtraction**	add sum-up subtract to subtract sth. (from sth.)	+ plus – minus	**addition and subtraction**
Multiplication **Division**	multiply to times sth. divide to divide sth. by sth.	• times ÷ divided by	**multiplication and division**

Addition and subtraction calculations — sum — difference

Calculation type	Numerical example	Example with variables
Only identical letters (variables) can be added or subtracted.	–	$27x + 3a - 5a + 3x - 9$ $= 27x + 3x + 3a - 5a - 9$ $= 30x - 2a - 9$
<u>Commutative law:</u> **Numbers and letters can be interchanged in their order**	5 + 8 – 6 – 7 + 17 = 17 + 5 + 8 – 6 – 7 = 30 – 13 = 17	$x + y - a + b + 3a$ $= 3a - a + b + x + y$ $= 2a + b + x + y$
<u>Associative law:</u> **Individual elements can be grouped into sub-totals**	5 + 8 – 4 + 3 = (5 + 8 + 3) – 4 = 16 – 4 = 12	$a + b - c + x$ $= (a + b + x) - c$

Obviously there are positive numbers, e.g. +8 or +12and negative numbers, such as -3 or -5. Also there are positive and negative coefficients of variables (letters) e.g. +y and -y.

<u>Furthermore:</u> The multiplication sign between a number and a variable (letter) or between variables can be omitted, for example, 5 • x = 5 x or a • b = ab. The coefficient 1 is generally not written, for example 1 • y = y. Elements may be grouped by parentheses, e.g. a + b - c = (a + b) - c.

The calculation rules for addition and subtraction:

Calculation rules	Numerical examples	Letter examples
Plus signs in front of brackets can be omitted. The signs of the individual elements remain unchanged.	25 + (8 – 6) = 25 + 8 – 6 = 27	$x + (y - z)$ $= x + y - z$
If there is a minus signs in front of a bracket, then the individual members within the bracket get the opposite sign when removing the bracket.	25 – (8 – 6) = 25 – 8 + 6 = 23	$x - (y - z)$ $= x - y + z$

Multiplication procedures

Calculation rules	Numerical example	Letter examples
Factor • Factor = Product	7 • 8 = 56	$a \cdot b = c$
Commutative law: The order of numbers and letters can be interchanged	7 • 8 • 4 = 8 • 4 • 7	$a \cdot b \cdot c = c \cdot a \cdot b$
Associative law: Individual elements can be grouped into sub-totals	7 • 4 • 8 = (7 • 4) • 8 = 8 • (7 • 4)	$x \cdot y \cdot z = (x \cdot y) \cdot z$ $= z \cdot (x \cdot y)$
The product of two factors of the same sign is positive: + times + = +; – times – = +	3 • 4 = 12 –3 • (–4) = 12	$a \cdot b = ab$ $-a \cdot (-b) = ab$
The product of two factors of unequal signs is negative: + time – = –; – time + = –	3 • (–4) = –12 –3 • 4 = –12	$a \cdot (-c) = -ac$ $(-a) \cdot c = -ac$
If a sum or difference in a bracket is multiplied by a factor, then each term of the sum or difference is multiplied individually by this factor. You also can calculate the bracket term (the sum or difference) first and then multiply the result by the factor subsequently.	8 • (3 + 4) = 8 • 3 + 8 • 4 = 56 or: 8 • (3 + 4) = 8 • 7 = 56 7 • (4 – 2) = 7 • 4 – 7 • 2 = 14 or: 7 • (4 – 2) = 7 • 2 = **14**	$a \cdot (x + 2x)$ $= ax + 2ax = 3ax$ or: $a \cdot (x + 2x)$ $= a \cdot 3x = 3ax$
If a bracket term (sum or difference) is multiplied by a further bracket term (sum or difference) then each member of the one bracket is multiplied by each member of the other bracket, taking into account the rules for signs. If **the** bracket terms (sum or difference) are numbers, then the bracket terms can also be calculated first. The product of the individual results gives the end result.	(10 – 7) • (3 + 5) = 10•3+10•5–7•3–7•5 = 30+50–21–35 = 24 or: (10 – 7) • (3 + 5) = 3 • 8 = **24**	$(a + b) \cdot (a - b)$ $= a \cdot a - a \cdot b + a \cdot b - b \cdot b$ $= a^2 - b^2$
	Note: a) Power notation = power mode, e.g. $a \cdot a = a^2$ b) (see **exponentiation**) b) individual members can neutralize each other when having different signs, i.e. the sum is zero: + 2 - 2 = 0 c) When dealing with more than two brackets, first of all the calculation of the two first brackets terms is performed. Then this result is processed with the next bracket term consecutively, etc.	

Division procedures

Calculation rules	Numerical examples	Letter examples
Numerator ÷ / : denominator = quotient	$12 \div 3 = \frac{12}{3} = 4$	$x \div y = \frac{x}{y} = c$
Numerator (dividend) and denominator (divisor) are not interchangeable. The commutative law does not apply.	$\frac{12}{3} \neq \frac{3}{12}$	$\frac{3x}{5a} \neq \frac{5a}{3x}$
A fraction line replaces a bracket, and vice versa.	$\frac{7+6}{y4} = (7 + 6) \div 4$	$\frac{x+y}{3} \bullet z = (x + y) \bullet \frac{z}{3}$
When dividing an expression (term) in brackets by a value (number, letter, parenthetical) each term in the parentheses is divided by this value. It is also possible first at all to calculate the term in brackets, and then divide it by the value.	$(20 - 5) \div 5$ $= 20 \div 5 - 5 \div 5$ $= 4 - 1 = 3$ **or:** $(20 - 5) \div 5$ $= 15 \div 5 = 3$	$(a + b) \div c = a \div c + b \div c$ $\frac{x-y}{x} = \frac{x}{x} - \frac{y}{x} = 1 - \frac{y}{x}$
If dividend and divisor are the same (equal), then the quotient = 1	$\frac{3}{3} = 1$	$\frac{a}{a} = 1$ $\frac{25x}{25x} = 1$
If each member of a term (sum or difference) is divisible by the same factor, then this factor can be factored out.	$36 + 12$ $= 6 \bullet 6 + 6 \bullet 2$ $= 6 \bullet (6 + 2)$ $= 6 \bullet 8 = \mathbf{48}$	$ax - ay$ $= a \bullet x - a \bullet y$ $= a \bullet (x - y)$
Numbers and letters are to be reduced as much as possible.	$\frac{16}{4} = \frac{4 \bullet 4}{4} = 4$	$\frac{49\,xy}{7y} = \frac{49 \bullet x \bullet y}{7y} = 7x$
If dividend and divisor have the same sign, then the quotient (the result) is positive. + Divided by + = + – Divided by – = +	$\frac{18}{6} = 18 \div 6 = 3$ $\frac{-18}{-6} = -18 \div (-)6 = 3$	$\frac{x}{y} = x \div y$ $\frac{-x}{-y} = \frac{x}{y} = x \div y$
If dividend and divisor have unequal sign, then the quotient (the result) is negative. + Divided by – = – – Divided by + = –	$\frac{18}{-6} = 18 \div (-6) = -3$ $\frac{-18}{6} = (-18) \div -6 = -3$	$\frac{x}{-y} = -\frac{x}{y}$ $\frac{-x}{y} = -\frac{x}{y}$
Note: Division (dividing) by zero is forbidden.		

Mixed calculation: (addition - subtraction) and (multiplication - division)

Calculation rules	Numerical examples	Letter examples
The sequence of solution steps is essential. The following procedure applies: multiplication and division first, then addition and subtraction.	$7 \bullet 6 - 4 \bullet 3$ $= 42 - 12 = 30$	$5x \bullet y - a \bullet 3b$ $= 5xy - 3ab$
	$= \frac{15}{5} + \frac{20}{4} - \frac{30}{6}$ $= 3 + 5 - 5 = 3$	$= \frac{15a}{3} - \frac{3x}{x} - \frac{8b}{2}$ $= 5a - 3 + 4b$

1.2 Calculation with fractions

A fraction consists of three symbols, namely numerator, fraction line and denominator: The fraction line can be replaced by a colon:
Fraction types:

Proper fraction < 1	Improper fraction > 1	Mixed number	Homonymous fractions	Unlike fractions	Apparent fraction (integers written as fractions)
Numerator < denominator	Numerator > denominator	Integer with fraction	Fractions with the same numerator	Fractions with different numerators	Numerator = 1
$\frac{3}{4}$	$\frac{7}{4}$	$\frac{7}{4} = 1 + = 1\frac{3}{4}$	$\frac{6}{7}, \frac{3}{7}, \frac{5}{7}$	$\frac{3}{4}, \frac{5}{7}, \frac{6}{9}$	$\frac{7}{1}, \frac{4}{1}, \frac{10}{1}$

Fractions can be reduced if necessary. The reverse calculation method is to expand fractions. However, the numerical value remains the same!

Expanding and reducing fractions

Calculation rules	Numerical examples	Letter examples
Expansion: multiply the numerator and denominator by the same number	$\frac{1}{3} = \frac{1 \cdot 6}{3 \cdot 6} = \frac{6}{18}$	$\frac{x}{y} = \frac{x \cdot a}{y \cdot a}$
Reduction: divide the numerator and denominator by the same number	$\frac{6}{18} = \frac{6 \div 6}{18 \div 6} = \frac{1}{3}$	$\frac{a \cdot x}{b \cdot x} = \frac{(a \cdot x) \div x}{(b \cdot x) \div x} = \frac{a}{b}$
Sums or differences in the numerator and denominator cannot be reduced. They have to be calculated before shortening or expanding.	$\frac{28-20}{7-5} = \frac{8}{2x} = 4$	$\frac{x-y}{y+x}$ can be expanded, but not be reduced

Addition and subtraction of fractions

Calculation rules	Numerical examples	Letter examples
When adding or subtracting homogeneous fractions the numerators are added or subtracted, but the denominator remains unchanged.	$\frac{6}{5} + \frac{3}{5} - \frac{1}{5} = \frac{6+3-1}{5} = \frac{8}{5} = 1\frac{3}{5}$	$-\frac{3}{x} + \frac{6}{x} + \frac{1}{x} - \frac{2}{x} = \frac{-3+6+1-2}{x} = \frac{2}{x}$
Unlike fractions have to have the same denominators before adding or subtracting the numerators; i.e. a common denominator has to be found, i.e. the lowest (possible) common denominator (LCD).	$\frac{3}{4} - \frac{1}{2} + \frac{1}{3}$ HN = 12 $= \frac{9}{12} - \frac{6}{12} + \frac{4}{12} = \frac{9-6+4}{12} = \frac{7}{12}$	$\frac{a}{x} + \frac{b}{y}$ HN = x • y $= \frac{y \cdot a}{x \cdot y} + \frac{x \cdot b}{x \cdot y} = \frac{y \cdot a + x \cdot b}{x \cdot y}$

The lowest common denominator (LCD) must be divisible by any single denominator without any remainder. The smallest number that satisfies this condition is called the least common multiple (LCM). Generally, the LCM can be easily recognized by simply looking at the numbers. But there is also a calculation method, namely splitting into prime factors, but this is not discussed further.

Note:	The product of all single denominators is a common denominator, but not necessarily the LCD.

Multiplication of fractions

Calculation rules	Numerical examples	Letter examples
Multiplication of an integer with a fraction: only the numerator is multiplied by the integer. The denominator remains unchanged.	$3 \bullet \frac{5}{6} = \frac{3 \bullet 5}{6} = \frac{15}{6} = 2\frac{3}{6}$ $= 2\frac{1}{2} = 2.5$	$3 \bullet \frac{x}{y} = \frac{6x}{y}$
Fraction times fraction: numerator times numerator and denominator times denominator.	$\frac{5}{6} \bullet \frac{4}{7} = \frac{5 \bullet 4}{6 \bullet 7} = \frac{20}{42} = \frac{10}{21}$	$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$
Integers or mixed fractions as a factor must first of all be converted into improper fractions or apparent fractions.	$3\frac{1}{3} \bullet 4 = \frac{10}{3} \bullet \frac{4}{1} = \frac{40}{3}$ $= 13\frac{1}{3}$	$2\frac{1}{2}$ a• 6b $= \frac{5a}{2} \bullet \frac{6b}{1}$ $= \frac{30ab}{2}$ = 15ab

Division of fractions

Calculation rules	Numerical examples	Letter examples
An <u>integer</u> is <u>divided by a fraction</u> by multiplying it with the reciprocal (inverse) fraction (exchange the numerator and denominator).	$7 \div \frac{3}{5} = 7 \bullet \frac{3}{5} = \frac{35}{3}$ $= 11\frac{2}{3}$	$5 \div \frac{a}{b} = 5 \bullet \frac{b}{a}$
A <u>fraction</u> is <u>divided by a fraction</u>, by means of multiplying one fraction with the reciprocal (inverse) fraction (exchange the numerator and denominator) of the other. Or if fraction-notation is used he numerator fraction is multiplied by the reciprocal (inverse) of the denominator fraction.	$\frac{5}{6} \div \frac{3}{4} = \frac{\frac{5}{6}}{\frac{3}{4}} = \frac{5}{6} \bullet \frac{4}{3}$ $= \frac{20}{18} = \frac{10}{9} = 1\frac{1}{9}$	$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \bullet \frac{d}{c}$ $= \frac{ad}{bc}$
Derived from the previous procedure: Fractions are divided by an integer by means of converting the integer into a fraction-notation and multiply the first fraction by the reciprocal of the second fraction.	$\frac{3}{5} \div 7 = \frac{3}{5} \div \frac{7}{1} = \frac{3}{5} \bullet \frac{1}{7}$ $= \frac{3}{35}$	$\frac{a}{b} \div 5 = \frac{a}{b} \div \frac{5}{1} = \frac{a}{b} \bullet \frac{1}{5}$ $= \frac{a}{5b}$

In decimal fractions (decimal numbers) integers or zeros are separated from the decimal places by a decimal point. For example; 0.5 or 3.7.

Converting fractions

Calculation rules	Numerical examples
A common fraction is converted to a decimal number by dividing the numerator by the denominator.	$\frac{6}{9} = 6 \div 9 = 0.\overline{6}$ infinitely periodic
A finite decimal fraction is converted to a common fraction, by just transferring all digits after the decimal point to the numerator. The denominator is written as a 1 followed by as many zeros as the number of digits of the numerator.	$0.674 = \frac{674}{1000} = \frac{337}{500}$

1.3 Powers and Roots

Additional special rules are required for calculations involving powers and roots. A product can consist of several identical factors, such as: a • a • a = a^3. The form a^3 is called the power notation.

$6^3 = 216$ ⍰ generally: $a^n = b$ $\quad a$ = **Base**

n = **Exponent**

b = **Value**

A power with a negative base is positive if the exponent is an even integer, and negative if the exponent is an odd integer.
For example: $(-6)^2 = (-6) \cdot (-6) = 36$ and $(-6)^3 = (-6) \cdot (-6) \cdot (-6) = -216$

A power that is in the denominator of a fraction can be written with a negative exponent in the numerator. Conversely, a power with a negative exponent in the numerator can be written as a power with a positive exponent in the denominator:

$\frac{1}{5^2} = 5^{-2}$ general: $\frac{1}{a^n} = a^{-n}$ or $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

Powers of base 10 are frequently used as shorthand notation for very small or very large numbers.

Prefix	Abbreviation	multiple of a unit
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	k	10^{3}
hecto-	h	10^{2}
deca-	da	10^{1}
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}

Exponentiation

Calculation rules	Numerical examples	Letter examples	Mathematical formula
Only powers of the same exponent and the same base may be added or subtracted.	$6^2 + 6^2 = 2 \bullet 6^2 = 72$ $\frac{3}{4^2} - \frac{2}{4^2} = \frac{1}{4^2} = \frac{1}{16}$ $= 4^{-2}$	$a^4 + 2a^4 = 3a^4$ $7x^n - 3x^n = 4x^n$	$ax^n + bx^n$ $= x^n$ (a + b)
When multiplying powers with the same base, the base is retained and the exponents are added.	$4^2 \bullet 4^3 = 4 \bullet 4 \bullet 4 \bullet 4 \bullet 4$ $= 4^5$ but also $4^2 \bullet 4^3 = 4^{2+3} = 4^5$	$a^3 \bullet a^3 = a^{2+3} = a^8$	$x^m \bullet x^n = x^{m+n}$
When multiplying powers with different base and the same exponent, the bases are multiplied and the exponent is maintained.	$4^2 \bullet 6^2 = (4 \bullet 6)^2$ $= 24^2 = 576$	$6x^2 \bullet 3y^2 = 18x^2y^2$ $= 18\ (x \bullet y)^2$	$x^n \bullet y^n = (x \bullet y)^n$
When dividing powers with the same base, the base is maintained and the exponents are subtracted.	$\frac{5^3}{5^2} = \frac{5 \bullet 5 \bullet 5}{5 \bullet 5} = 5$ but also $5^3 \div 5^2 = 5^{3-2}$ $= 5^1 = 5$	$\frac{a^3}{a^2} = \frac{a \bullet a \bullet a}{a \bullet a} =$ a but also $a^3 \div a^2 = a^{3-2}$ $= a^1 =$ a	$\frac{x^m}{x^n} = x^{m-n}$
Powers with the same exponent are divided by dividing their bases and maintaining the exponent.	$\frac{16^2}{4^2} = (\frac{16}{4})^2 = 4^2 = 16$	$\frac{m^3}{n^3} = (\frac{m}{n})^2$	$\frac{a^n}{b^n} = (\frac{a}{b})^n$
When multiplying powers by a factor, first the power must be calculated.	$5 \bullet 8^3 = 5 \bullet 512 = 2{,}560$ or $14 \bullet 10^{-2} = \frac{14}{10^2} = 0.14$	—	—
A power with exponent zero has the value 1	$\frac{15^3}{15^3} = 15^{3-3} = 15^0 = 1$	$(x+y)^0 = 1$	$a^0 = 1$

Taking roots is the inverse function of exponentiation.

$\sqrt[2]{25} = 5$ general: $\sqrt[n]{a}$ = b

n = **degree of a root or index**
a = **radicand**
b = **root value**

In most cases the index of a square root is not written. Hence it is: $\sqrt[2]{a} = \sqrt{a}$
For cube roots the index is 3, for example $\sqrt[3]{17.5^3} = 17.5$

Taking roots

Calculation rules	Numerical examples	Letter examples	Mathematical formula
A root calculation also can be represented in power notation. The radicand gets a fraction as an exponent: numerator = exponent of the radicand, denominator = root index	$\sqrt{7^4} = \sqrt[2]{7^4}$ $= 7^{\frac{4}{2}} = 7^2 = 49$	$\sqrt[3]{a^5} = a^{\frac{5}{3}} = a^{1.6\overline{6}}$	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$ special case: $\sqrt[n]{a} = \sqrt[n]{a^{-1}} = a^{\frac{1}{n}}$
Roots can only be added and subtracted if they have the same exponent and radicand. The coefficients are added or subtracted and the roots are preserved	$2 \bullet \sqrt{7} + 5 \bullet \sqrt{7}$ $= (2 + 5) \bullet \sqrt{7}$ $= 7 \bullet \sqrt{7}$	$6 \bullet \sqrt{a} - 8 \bullet \sqrt{a}$ $= -2 \bullet \sqrt{a}$	$a \bullet \sqrt{m} + b \bullet \sqrt{m}$ $= (a + b) \sqrt{m}$
The square root of a product can be calculated from either the product or the factors	$\sqrt{16 \bullet 25} = \sqrt{400} = 20$ but also $\sqrt{16 \bullet 25} = \sqrt{16} \bullet \sqrt{25}$ $= 4 \bullet 5 = 20$	$\sqrt[5]{a \bullet b} = \sqrt[5]{a} \bullet \sqrt[5]{b}$	$\sqrt[n]{a \bullet b} = \sqrt[n]{a} \bullet \sqrt[n]{b}$
If the radicand is a sum or difference, first of all, the sum or difference has to be calculated. Then the root can be calculated.	$\sqrt{16 + 9} = \sqrt{25} = 5$ or $\sqrt{3^2 - 2^3} = \sqrt{9 - 8}$ $= \sqrt{1} = 1$	$\sqrt[3]{x - y} = \sqrt[3]{(x - y)}$	$\sqrt[n]{(a - b)} = \sqrt[n]{(a - b)}$ $\sqrt[n]{(a + b)} = \sqrt[n]{(a + b)}$
In case the radicand is a fraction (ratio), then the root can be calculated from the quotient or individually from the numerator and denominator.	$\sqrt{\frac{16}{25}} = \sqrt{0.64} = 0.8$ but also $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5} = 0.8$	$\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

1.4 Theory of equations and formulas

An **equation**, in a mathematical context, is generally understood to mean a mathematical statement that asserts the equality of two expressions. In modern notation, this is written by placing the expressions on either side of an equals sign (=), for example: $x + 3 = 5$.

Equation types

Type of equation	key example
Equations relating physical quantities	$P = \frac{F \cdot s}{t}$ or $a = \frac{\Delta v}{\Delta t}$ (see physics)
Numerical equations represent statements about the relationship between numbers and quantities. They apply only when the quantities are used in the right units. These should be use only in special cases.	$M = 9550 \cdot \frac{P}{n}$ only valid in case of: *M* = moment of a torque [Nm] *P* = power [kW] *n* = rotation speed [min^{-1}]
Conditional equations (algebraic equations) are used to compute values of variables.	$x + 27 = 45$ x is the variable $x = 45 - 27$ $x = 18$

Note: Manipulating equations must always be based on the principle that a change on one side of the equation must result in maintaining the identity. Hence the other side has to be modified in the same way.

Rearranging equations

Type of calculation	Example (rearrange by x)
Addition	11x + 7 – 5a = 4x + 9 + 3a | -4x + 5a – 7 11x + 7 – 5a – 4x + 5a – 7 = 4x + 9 + 3a – 4x + 5a – 7 7x = 2 + 8a | ÷ 7 x = $\frac{2+8a}{7}$
Subtraction	7.1x – 12.79 = 5.8 – 4.8x – 2.4x 7.1x – 12.79 = 5.8 – 7.2 | +12.79 + 7.2x 7.1x – 12.79 + 12.79 + 7.2x = 5.8 – 7.2x + 12.79 + 7.2x 14.3x = 18.59 | ÷ 14.3 x = $\frac{39}{9} = 4\frac{3}{9} = 4\frac{1}{3}$
Multiplication	9 (3x – 11) = 6 (3x – 10) 27x – 99 = 18x – 60 | +99 – 18x 27x – 99 + 99 – 18x = 18x – 60 + 99 – 18x 9x = 39 | ÷ 9 X = $\frac{39}{9} = 4\frac{3}{9} = 4\frac{1}{3}$
Division	a = c + $\frac{x}{b}$ | swap sides c + $\frac{x}{b}$ = a | -c $\frac{x}{b}$ = a – c | • b x = b (a – c)
Exponentiation	1. a (x - b^2) = b (x - a^2) ax - ab^2 = bx - ba^2 | -b + ab^2 ax - ab^2 - bx + ab^2 = bx - ba^2 – bx + ab^2 ax - bx = ab^2 - ba^2 | ÷ (a • b) x = $\frac{ab^2 - ba^2}{a-b}$ 2. $\sqrt{x}$ + a = 60 | -a $\sqrt{x}$ = 60 – a | $( )^2$ x = $(60 – a)^2$
Taking roots	x^3 = 60 | $\sqrt[3]{\ }$ x = 4

2 STATISTICAL METHODS

2.1 Basics of statistics

2.1.1 Where does statistics come from – what is statistics?

The beginnings of the statistics can be found in censuses before and at the beginning of our common era. However, in the 18th century, it began to develop as an independent scientific discipline. It was used to describe the features that characterize the situation of a country. The term statistics was derived from the Latin word status which means the state and/or situation. The methods of mathematical statistics have become an effective tool in science and technology in the detection of phenomena that conform to the laws of nature.

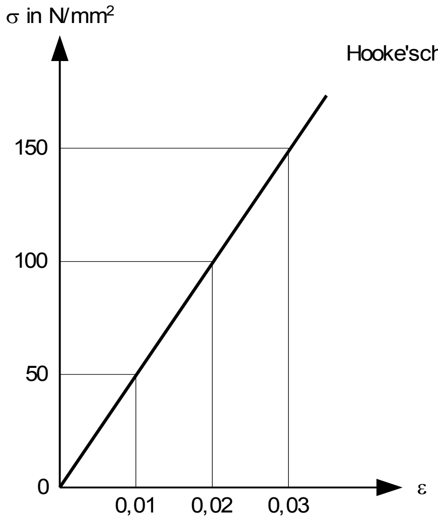
Note:	Statistics is a tool for studying mass phenomena and high volume transactions to be numerically quantified.

It has its mathematical foundation in the law of large numbers, because if the number of individual case reports is sufficiently large, the random deviations of the individual cases are insignificant and the typical characteristics are revealed.

Note:	The law of large numbers requires sufficient statistical data material. Drawing a conclusion about events in general from an individual survey requires that the data was collected according to meaningful criteria.

2.1.2 The tools of statistics

Spread sheets, charts, and monograms are used for the brief, well-arranged, and thus quick collection and presentation of data.
Numbers within tables may also be in a functional relationship, which can be represented graphically in a diagram.



Extension: $\varepsilon = \frac{1}{E} \bullet \sigma$ $\varepsilon = \frac{\Delta l}{l_0}$

Δl = change in length
l_0 = initial length

If there is a functional relationship, the value associated with it can be calculated from given or estimated data.
The method mentioned above is also widespread in mathematics, representing functions graphically. Consider the function

$y = 3x - 2$

Then for every value of x there is a unique value of y. For example, when x = 10 then y = 28, when x = -10 then y = -32. The table of values for the range of x = -2 and x = 5 yields the following results:

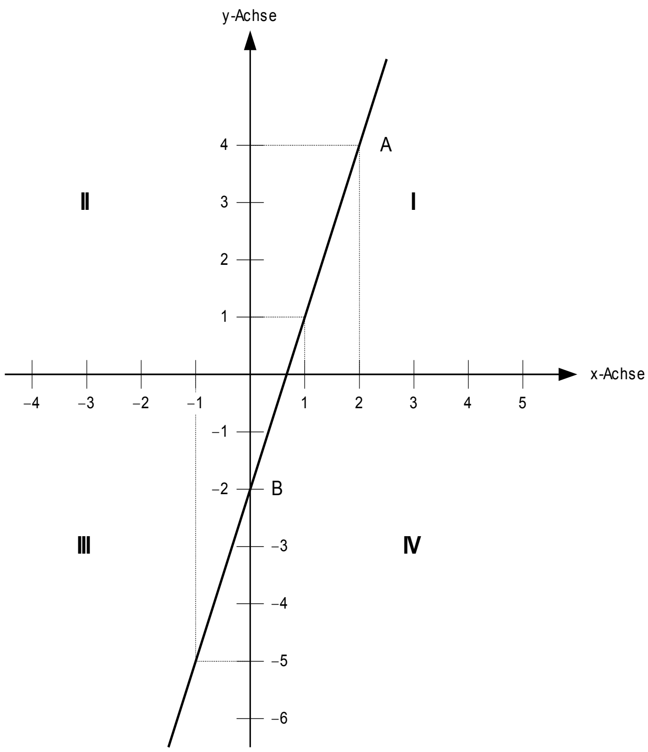
x	2	1	0	1	2	3	4	5
y	8	5	2	1	4	7	10	13

To be able to represent the curve of the function, two lines that intersect at right angles are used, called the Cartesian coordinate system. The graphical representation of the function

$y = 3x - 2$ with the above table of values is shown in the figure below.

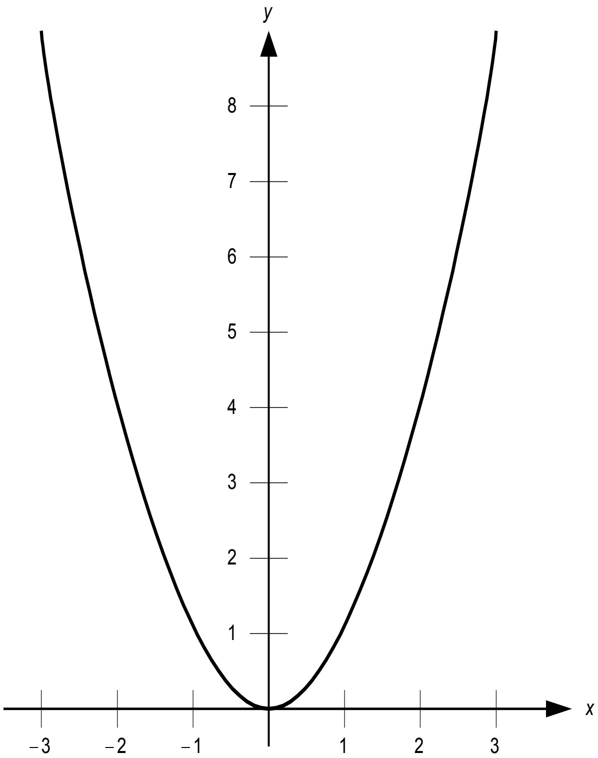
The intersection of these two lines, which are called coordinate axes or just axes, is the zero-point or origin. The horizontal axis is called the x-axis or abscissa. The vertical axis is called y-axis or ordinate. The projections onto the axes are known as the coordinates. Four areas are generated by both axes, which are called the quadrants:

Quadrant I (1. Quadrant): $+x$, $+y$
Quadrant II (2. Quadrant): $-x$, $+y$
Quadrant III (3. Quadrant): $-x$, $-y$
Quadrant IV (4. Quadrant): $+x$, $-y$



For the function $y = x^2$ (normal parabola), the following table gives the values in the range x = -3 to x = +3. The associated graphical representation in the coordinate system is shown as follows:

x	3	2	1	0	1	2	3
y	9	4	1	0	1	4	9



Note:	The Cartesian coordinate system provides the possibility to show the location of any point (x,y) in the x, y plane.

Nomograms represent relationships between several variables as easily interpretable graphical images.

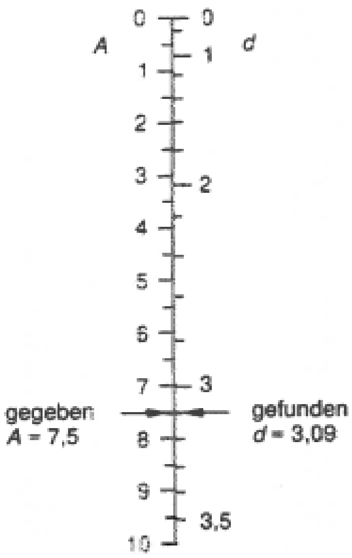
The function

$$A = \frac{\pi}{4} \bullet d^2$$

is to be displayed in a nomogram. First of all, a table of values is generated:

d in cm	0	1	2	3	4	5	6	7	8	9	10
A in cm^2	0	0.78	3.14	7.07	12.57	19.64	28.27	38.48	50.27	63.62	78.54

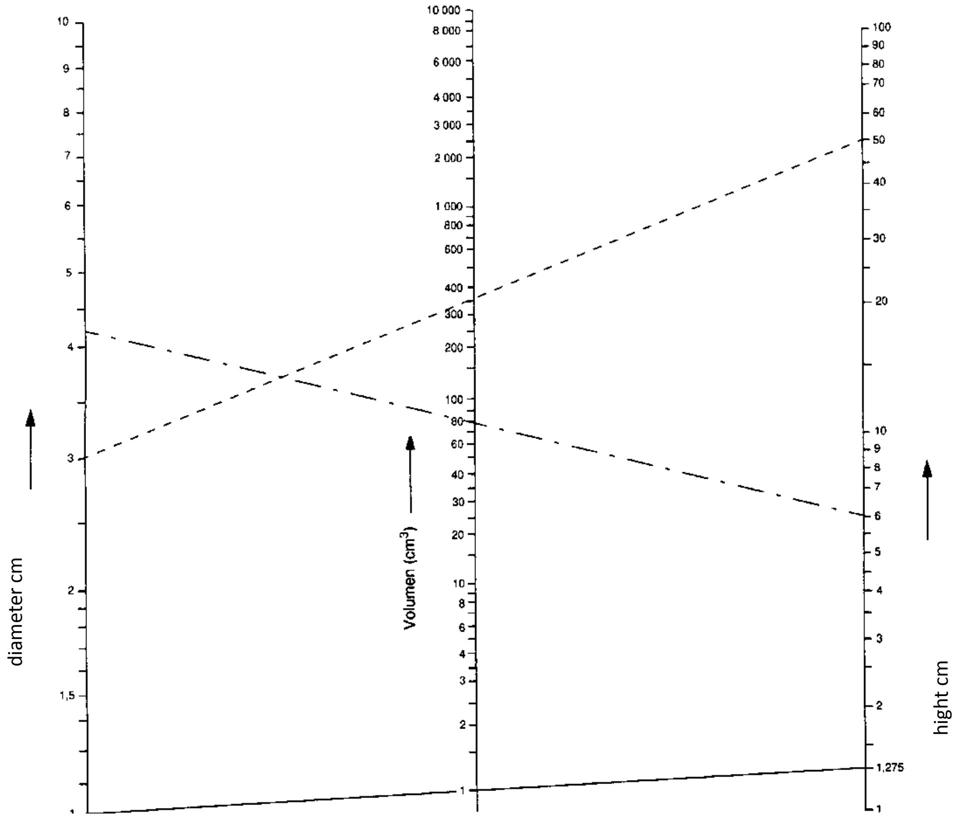
Of course, instead of using Cartesian coordinates, the two axes can also be drawn parallel to one another or indeed both onto one vertical line with the A-scale the left and d scale right side. This is shown in the figure below:



In the same manner, nomograms for even three or more variables are possible. This is the case for the following function:

$V = \frac{\pi}{4} \bullet d^2 \bullet 1$

With this formula, the volume of a cylinder is calculated, where d is the cylinder diameter and l is the length of the cylinder or the cylinder height.



The volume of a cylinder is determined using the following parameters:

a) Diameter *d* = 3 cm and height *l* = 50 cm

b) Diameter *d* = 4.2 cm and height *l* = 6 cm

Result:

a) $V = 350\ cm^3$ (dashed line)

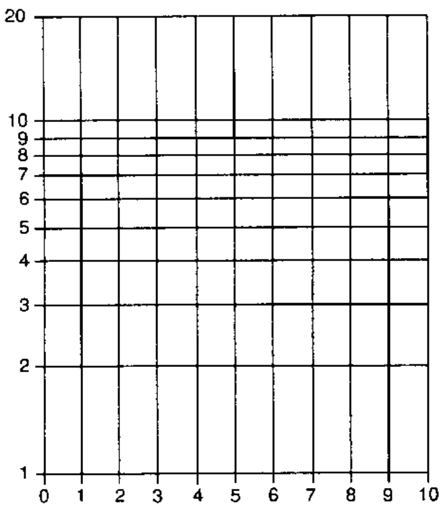
b) $V = 80\ cm^3$ (dot and dash line)

Note:	Charts and nomograms have the undoubted advantage to provide quick results. However, unfortunately inaccuracies are unavoidable due to some reading errors which always occur.

Sometimes it is most convenient that the divisions of the scales may not be arranged arithmetically; in some cases it is necessary to use non-linear graduation.

Very often, what is known as logarithmic scales are used. This may apply to one or more (even all) scales (axes), the other axes can be graded linearly or quite differently depending on the relevant application(s).

Example of a semi-logarithmic grid:



Functions can also be represented in such kind of grids or even when using double-logarithmic axes.

Usually dependencies between three variables are illustrated in such kind of graphs, (**grid table**) for example the function:

$v_c = \pi \bullet d \bullet n$

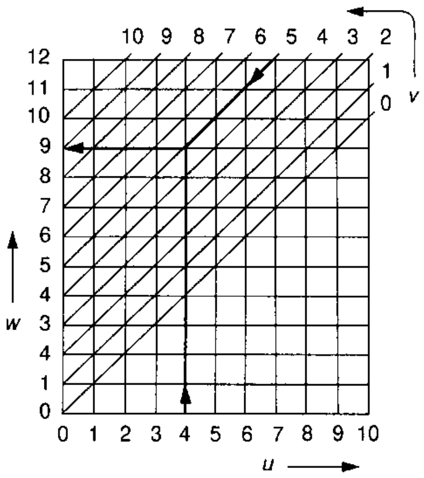
v_c = cutting speed [m/min]

d = workpiece diameter [m]

n = rotation speed [min^{-1}]

The corresponding **grid table** is referred to as a rotation speed diagram.

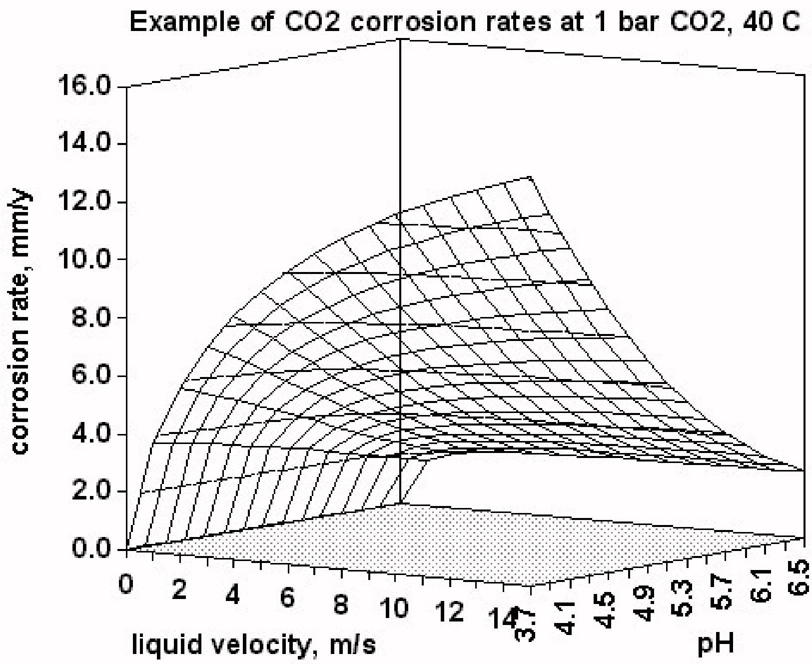
The graph below shows the structure of such a **grid table**, but in this case both axes have a linear scale.



Determination of the number w, if the numbers u = 4 and v = 5 are given.
Result: *w* = 9 (follow bold-type lines)

Such grid tables are available for all kinds of arithmetic operations in mathematics and technological problems!

Example: CO_2-Corrosion:



2.2 Planning tests and collecting data

2.2.1 Overall population and samples

In statistics a set of observations or surveys under the same criteria and conditions is called a population.
Every single test or individual observation or measurement is an element of the population.

Each element can be examined regarding various features which can be interpreted as random variables X, Y,...,.A finite subset of elements from the population is always considered in case studies. It is called a sample, and the number N of elements involved is called the sample size.

In statistics, large and small samples sizes are differentiated:
small sample size -> $N \leq 30$ items
large sample size -> $N > 30$ elements

2.2.2 Experimental design and raw data

The first step in using statistical methods to solve a problem is to design the experiment. The plan should include how the data will be collected, the sample size and how the problem will be solved. Good formulation of the evaluation procedure will yield satisfactory deliverables. The following considerations are important for an evaluation of a test process:

a) The material investigated has to be homogeneous, i.e., the experimental method must remain the same, equipment and production conditions must not change, only measuring instruments with the same accuracy may be used.
b) Systematic errors must be eliminated. For example: If a statement about the suitability of two materials for a production process is wanted, each material must processed on the same type of machine tool.
c) Values for control groups must be provided, which means, for example, that in fertilizer experiments, the difference between fertilized and unfertilized plants growing under the same environmental conditions, is detected.
d) The sample must be randomly or representatively selected. A selection is only random if the probability that it belongs to the sample is the same for each element. When testing a consignment of screws, the sample must not contain just screws from one part of the consignment but must rather contain screws from the whole batch.
e) The sample size must be appropriate. The larger the volume of available data, (i.e. the greater the number of elements, N), the better the confidence level of the statistical evidence that can be achieved.

When collecting the statistical data, the data acquired and the observations are mainly processed using computerized systems and will finally be recorded in lists and/or files.
The whole collection of unprocessed data is referred to as the raw data.

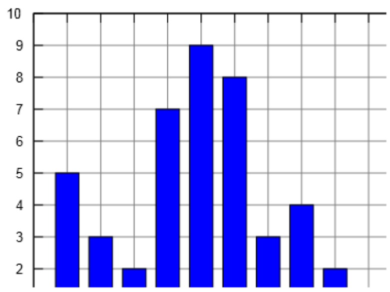
2.3 Evaluation of the data

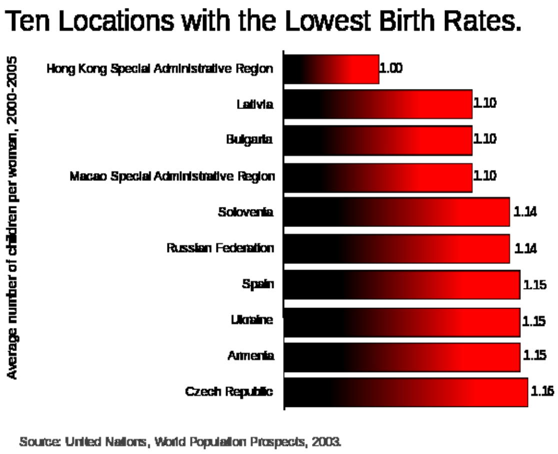
2.3.1 Distribution sheets

Before one can get an overview of the data, one must prepare it. This means that the raw data about characteristics and characteristic values is arranged according to its size and the frequency with which it occurs is determined. This gives the frequency distribution.

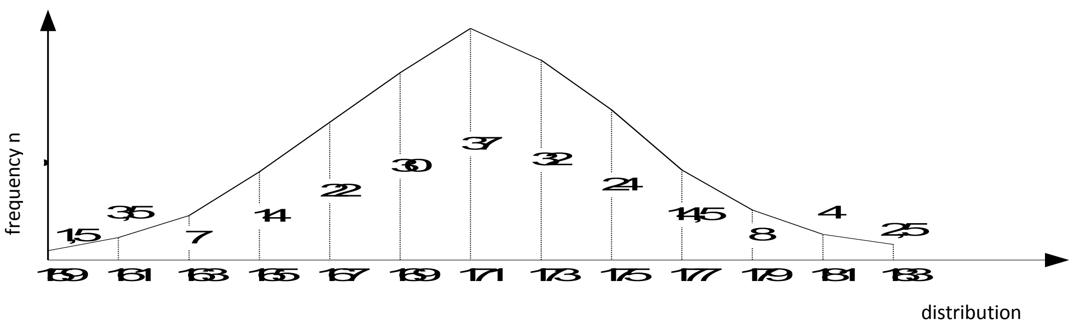
2.3.2 Representing frequency distributions graphically

Frequency distributions according to classes are shown in graphs with a rectangular coordinate system. On the horizontal axis, the abscissa, the classification is represented by lines or bars using an appropriate scale (frequency distribution vertical-bar graph).
In the rectangular bar representation (figure below) of each class, a rectangle (bar) width is specified. Its height is equal to the frequency N of the class in a fitting scale.
Examples of possible bar graphs representing statistical data:





A polygon representation is obtained from a square representation, by drawing lines along the upper centres of the rectangle bars.



Both representations, the polygonal representation and the bar graph, indicate the frequency distribution very well.

2.3.3 Arithmetic Mean

The clear and precise grouping of the distribution sheet and the clear graphical presentation describe a sample excellently. Nevertheless it is common in statistics to go a step further. We want to reduce a long series of numbers (e.g. from 200 or even more elements) to only a few numbers which indicate the situation precisely and accurately. This is done most effectively using the arithmetic mean, which is denoted by the character $\bar{x}$ and formula.

$$\bar{x}_{arithm} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

It is calculated from all the values of the random samples (i.e. characteristic values) by adding them and dividing the sum by the number of sample elements N (which is also known as the sample size or lot size). The calculation can be simplified by noting that an individual value, x, can occur several times, i.e. with frequency n:

To determine the sum of all the characteristic values, x, each characteristic value is multiplied by its frequency n and all these products are added.

Dividing this sum by the number of the sample elements finally results in the mathematical form of the formula for the arithmetic mean (arithmetic average):

$$\bar{x} = \frac{n_1 + x_1 + n_2 \bullet x_2 + n_3 \bullet x_3 + \cdots}{N}$$

The summation sign Σ and a general index i are usually used to describe such operations. Thus, the arithmetic mean is:

$$\bar{x} = \frac{\Sigma(n_i \bullet x_i)}{N}$$

In addition to this, another notation – frequently found in the literature – is explained for the arithmetic mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

n = number of samples = N

Do not confuse this "n" with the occurrence frequency "n "!

Example (applying two different kinds of calculation):

A spindle with a diameter d = 20 mm is produced in large quantities. There are five pieces taken for statistical analysis with diameters of:

d_1 = 20.01 mm
d_2 = 20.05 mm
d_3 = 19.78 mm
d_4 = 19.99 mm
d_5 = 20.02 mm.

a) N = 5, occurrence frequencies $n_1 = n_2 = n_3 = n_4 = n_5 = 1$

$$\bar{x} = \frac{\Sigma(n_i \cdot x_i)}{N} = \frac{n_1 \cdot d_1 + n_2 \cdot d + n_3 \cdot d_3 + n_4 \cdot d_4 + n_5 \cdot d_5}{N}$$

$$\bar{x} = \frac{1 \cdot 20.1\,mm + 1 \cdot 20.05\,mm + 1 \cdot 19.78\,mm + 1 \cdot 19.99\,mm + 1 \cdot 20.02\,mm}{5}$$

$$\bar{x} = \frac{99.94\,mm}{5} = 19.988 \text{ mm}$$

b) $n = 5 \notin N$ (number of elements or measurements)

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} d_i = \frac{1}{5} \cdot (20.01 \text{ mm} + 20.05 \text{ mm} + 19.78 \text{ mm} + 19.99 \text{ mm} + 20.02 \text{ mm})$$

$$\bar{x} = \frac{1}{5} \cdot 99.94 \text{ mm} = 19.988 \text{ mm}$$

2.3.4 Range, standard deviation and variance

Each data-set of samples has a minimum and a maximum value. The range R of the samples, which is also known as variation, is the difference between the minimum and maximum values. Range:

$R = x_{max} - x_{min}$

<u>Example:</u> Calculation of range for the shafts above

$R = d_{max} - d_{min}$ = 20,05 mm – 19,78 mm

<u>*R* = 0,27 mm</u>

Assuming the shafts have to be made with h6 tolerance. Using a fitting table the dimensions can be determined.

<u>Tolerances:</u>

upper dimension: *es* = 0,
lower dimension: ei = -13 μm

Thus the **maximum dimension** of the shaft is: G_{oW} = 20 mm,
the **minimum dimension** of the shaft is: G_{uW} = 19.984 mm

Since the calculated mean dimension = 19.988 mm, and according to this statistical quality control one might feel that the production process is proper. But this is not the case, as can be seen in the test samples. Only one of the samples with diameter d_4 = 19.99 mm is within tolerance zone. It can be seen:

Note:	Arithmetic means are only a reliable criterion if the random variables x, i.e. the measured values, are not scattered too much around the mean.

In order to assess the facts properly, the standard deviation s, and the variance s^2 are used as additional evaluation criteria.

Note:	The characteristic statistic parameters standard deviation s and variance s^2 help to determine the variation of the measured values.

Because of the complex mathematical facts – the derivation of the corresponding equations is skipped. The illustration and explanation of the formulas and formula parameters will be sufficient.

Standard Deviation: $s = \sqrt{\frac{1}{n} \bullet \sum_{i=1}^{n}(x_i - \bar{x})^2}$

n = number of measurements
x_i = value measured (data)
$\bar{x}$ = arithmetic mean

The $(x_i - \bar{x})^2$ terms are called the deviation squares. Thus:

Note:	The standard deviation is the deviation of the values measured from the arithmetic mean. It is calculated by computing the root of the product of the reciprocal of the number of samples and the sum of deviation squares.

Using the data of the earlier example above the calculation is shown below:

$(d_1 - \bar{x})^2$ = (20,01 mm – 19,988 mm)^2 = (0,022 mm)^2 = 0,000484 mm^2
$(d_2 - \bar{x})^2$ = (20,05 mm – 19,988 mm)^2 = (0,062 mm)^2 = 0,003844 mm^2
$(d_3 - \bar{x})^2$ = (19,78 mm – 19,988 mm)^2 = (–0,0208 mm)^2 = 0,043264 mm^2
$(d_4 - \bar{x})^2$ = (19,99 mm – 19,988 mm)^2 = (0,002 mm)^2 = 0,000004 mm^2
$(d_5 - \bar{x})^2$ = (20,02 mm – 19,988 mm)^2 = (0,032 mm)^2 = 0,001024 mm^2

$\sum_{i=1}^{n}(x_i - \bar{x})^2$ = 0.04862 mm^2

$s = \sqrt{\frac{1}{n} \bullet \sum_{i=1}^{n}(x_i - \bar{x})^2}$ = s = SQR $\sqrt{(\frac{1}{5} \bullet 0.04862)} = 0.0986$ mm

Another test used frequently for determining the deviation of the values measured from the arith-

metic mean is the variance $s^2 = \sqrt{\frac{1}{n-1} \bullet \sum_{i=1}^{n}(x_i - \bar{x})^2}$

Taking the square root from this expression results in the corrected standard deviation σ:

$$\sigma = \sqrt{\frac{1}{n-1} \bullet \sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Unlike the uncorrected standard deviation s, σ in the corrected standard deviation is calculated with n - 1 in the denominator of the root expression.

Let us calculate the corrected standard deviation of the values used before:

$$\sigma = \sqrt{\frac{1}{n-1} \bullet \sum_{i=1}^{n}(x_i - \bar{x})^2} = \sqrt{\frac{1}{5-1} \bullet 0.04862 \text{ mm}^2} = \sqrt{\frac{0.04862\ mm^2}{4}}$$

σ = 0.1102 mm^2

Comparing the equations for the standard deviation and the true standard deviation, shows:

Note:	At sufficiently large sample sizes (e.g., N = 1 000) we have that: $s \approx \sigma$

2.3.5 Statistical confidence

Note:	The confidence level, P, is the probability that a measurement is sufficiently close to the arithmetic mean. The corresponding interval around the mean is called the confidence interval.

Various technical areas have a preferred confidence level.

In production technology the preferred confidence level is P = 95%.

Equations have been developed to calculate the true mean μ depending on of the confidence level P.

For the **admissible** true mean at P = 95% we have:

$$\mu = \bar{x} \pm \frac{2 \bullet \sigma}{\sqrt{n}}$$

The true mean can be calculated by means of the coefficient of safety t, which depends on the sample size N, i.e. the number of elements (measurement values):

Actual true mean at P = 95%:

$$\mu = \bar{x} \pm \frac{t}{\sqrt{n}} \bullet s$$

Coefficient of safety at P = 95%:

Sample size *n*	3	4	5	6	8	10	20	30	50	100	200	more than 200
Coefficient of safety *t*	4.3	3.2	2.8	2.6	2.4	2.3	2.1	2.05	2.0	1.99	1.97	1.96

Using the data from the earlier example gives us the following:

a)

$\mu = \bar{x} \pm \frac{2 \bullet \sigma}{\sqrt{n}}$ = 19.988 mm $\pm \frac{2 \bullet 0.1102\ \text{mm}}{\sqrt{5}}$

μ = 19.988 mm ± 0.0986 mm → μ_1 = 20,0866 mm

μ_2 = 19.8894 mm

b)

$\mu = \bar{x} \pm \frac{t}{\sqrt{n}} \bullet$ s   t = 2.8 bei n = 5

μ = 19.988 mm $\pm \frac{2.8}{\sqrt{5}} \bullet$ 0.0986 mm

μ = 19.988 mm $\pm$ 0.1234 mm → μ_1 = 20,1114 mm

μ_2 = 19.8894 mm

From the results of a) and b) it can be seen that P = 95% is not reached.

3. THE TECHNICAL USES OFF STATISTICS

3.1 Ability and capability indices

Note:	A manufacturing process is known as capable if it is possible to ensure the **tolerance requirements of the quality criteria monitored x.**

Note:	Measurement capability describes the suitability of a measuring instrument for obtaining reliable results.

Note:	In technical statistics, the capability of both the processes and the measuring equipment is required.

3.2 Statistical process control using control charts

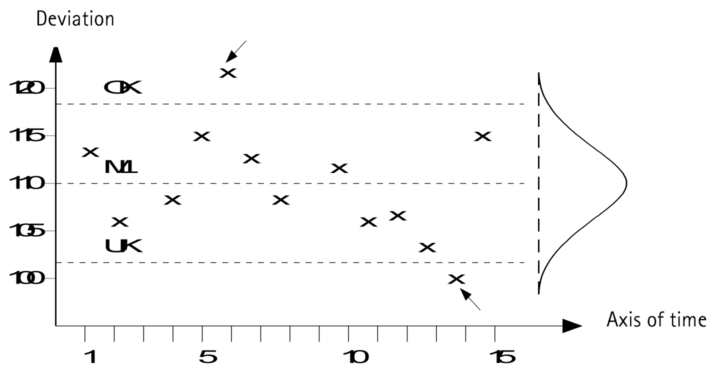
Statistical methods are used to monitor the quality of production process. Statistical quality control is based on monitoring the production process using statistical methods so that deficient products are recognized at the moment of generation and the problems causing this can be remedied as early as possible. Production is examined using control charts to ensure that waste or extra effort required for reworking can significantly be reduced or even be avoided.

a) Finished products and semi-finished products are examined on the basis of statistical sampling plans to determine whether they meet the quality standards. Here, production cannot be influenced straightaway. However, conclusions for the subsequent production can be obtained.

Note:	By means of control charts, the production process can be influenced directly (on-line).

The manufacturing process is monitored by testing the items produced for characteristic product features, using control charts. In case of deviations from specified reference values it can be decided whether intervening in the manufacturing process is necessary or not.

We explain the structure and use of control charts via individual sample charts.

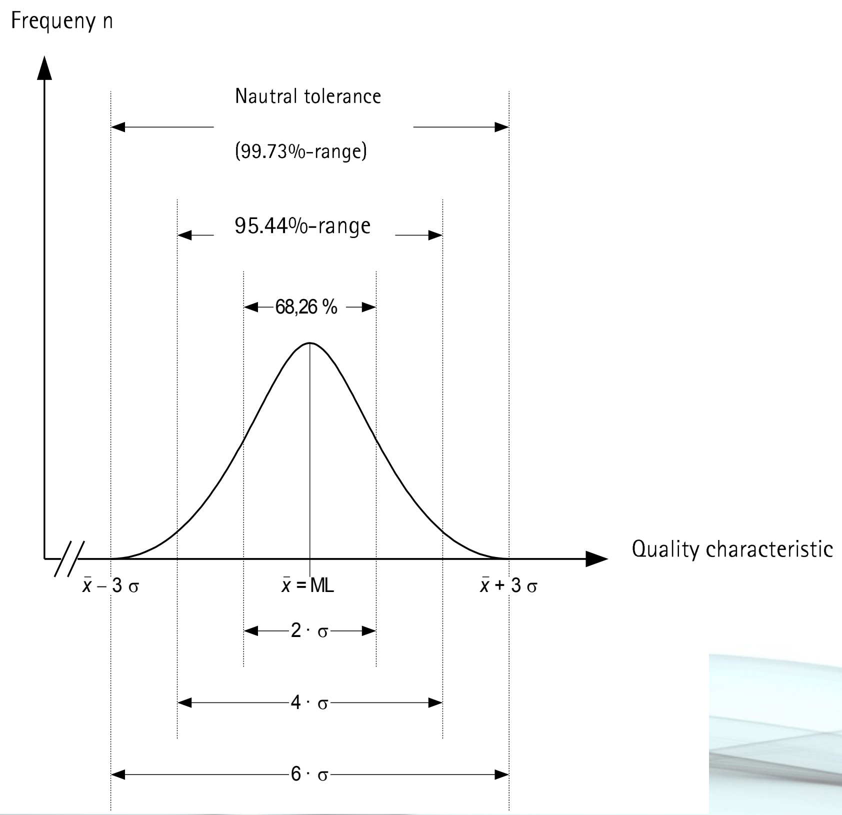


Suppose we are monitoring a characteristic – for example, a dimension. We divide time into intervals and for each interval we choose a piece produced in it randomly. We measure the characteristic. This dimension measurement is entered on the control chart with the appropriate interval or test numbers.

From the entries plotted we have a frequency distribution. In many cases a so-called normal distribution is obtained, at least approximately.

This can be seen on the right side of the figure above as a trend, and it is also recognizable in some earlier figures of this script. Explanation in more detail is given before the use of the control chart is explained:

Normal distribution (Gaussian plot)



The function (graph) of the normal distribution was deduced by the German mathematician Gauss using probability theory. It can be seen that most samples are near $\bar{x}$ = ML. To the right or left there are fewer samples. If σ is regarded to be the true standard deviation, then following fractions of values are within certain limits:

68.26 % are in the range $\bar{x} \pm \sigma$
95.44 % are in the range $\bar{x} \pm 2\,\sigma$
99.73 % are in the range $\bar{x} \pm 3\,\sigma$ (natural tolerance)

Technology requires confidence levels of P = 95%, i.e. a range (interval) of $\bar{x} \pm 2\,\sigma$. This interval is called a confidence interval, see above.

Applied to the above shaft example, this means:
19.988 mm $\pm$ 0.1102 mm → *limit*$_1$ (V_1) = 20.0982 mm
limit$_2$ (V_2) = 19.8778 mm

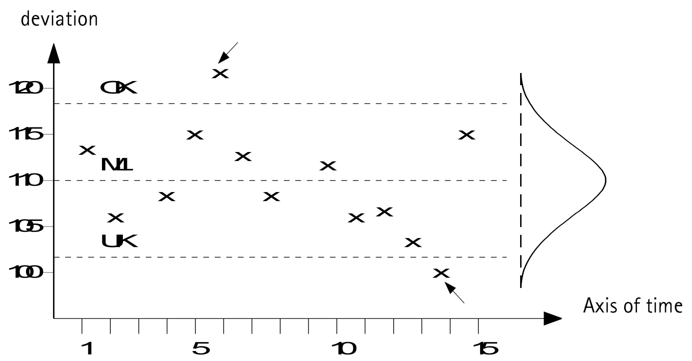
Let us use look at the graph again. The confidence interval for the confidence level of P = 95.44% is shown. This degree of accuracy, for example, is assumed in scientific statistics.

The mean centre line is marked as ML, and taking an interval with limits
$\bar{x} + 3\,\sigma$ as upper control line OK and
$\bar{x} - 3\,\sigma$ as lower control line UK

then the area under the line in this interval includes 99.73% of all measurements. However this is only valid if a perfect Gaussian distribution can be assumed.
To monitor the process with the control chart, the centre line ML =$\bar{x}$, the upper control line OK and the lower control line UK need to be fixed.

If a value is measured which is located in the region between upper and lower control limit, then the deviation from the average value $\bar{x}$ is considered to be random.

The deviation is significant if a value is located outside the control lines (indicated by an arrow in the Figure below). In this case the reason for faults has to be identified before production can continue.



The control chart shows the evolution of the feature observed during the production process.

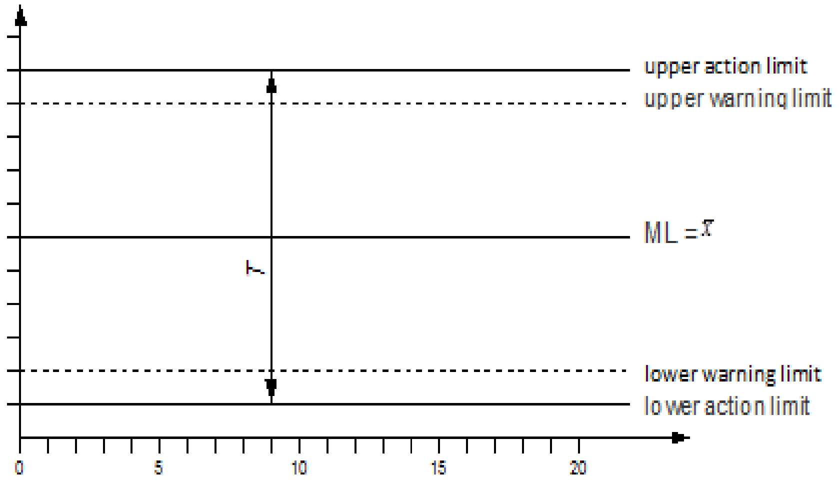
The makes a control chart very helpful. In combination with good trouble shooting capabilities, this can also have a lasting effect as an indicator of recurrent faults by showing when technology needs to be changed. This improves the capability of the process.

3.3 Calculation of capability indices

Control charts can be used to influence quality by permanent and immediate visualization of the actual quality; generally known as:

Quality Control Charts (QCC)

We have already covered terms like upper control line and lower control line. The Figure below illustrates a schematic QCC with names, currently used in production:



T = (**U**pper **A**ction **L**imit - **L**ower **A**ction **L**imit) = UAL – LAL
Process width:

$P_\sigma = \pm 3 \bullet \sigma = 6\,\sigma \equiv \quad P = 99{,}73\ \%$

In manufacturing, many different capability indices are used - too many to explain them all. The notation C is, however, common to all of them. We will only deal with the C_p index:

C_p-Index: $C_p = \frac{T}{P_\sigma} = \frac{UAL-LAL}{P_\sigma}$

P [%]	Range (Process spread)	Capability index C_p
68.26	$\pm\ \sigma \notin\ 2\,\sigma$	0.33
95.44	$\pm 2\,\sigma \notin\ 4\,\sigma$	0.67
99.73	$\pm 3\,\sigma \notin\ 6\,\sigma$	1.0
99.9937	$\pm 4\,\sigma \notin\ 8\,\sigma$	1.33
99.99994	$\pm 5\,\sigma \notin\ 10\,\sigma$	1.67
99.9999998	$\pm 6\,\sigma \notin\ 12\,\sigma$	2.0

Example:

d_1 = 20.01 mm
d_2 = 20.05 mm
d_3 = 19.78 mm
d_4 = 19.99 mm
d_5 = 20.02 mm

The mean is $\bar{x}$ = 19.988 mm. Hence the corrected standard deviation is σ = 0.1102 mm. What is the tolerance range T for a capability index C_p = 1.0?

$C_p = \frac{T}{P_\sigma}$ = OSG – USG = $C_p \bullet P_\sigma$ = 1.0 • 6 • 0.1102 mm

T = 0.6612 mm

Note:	Finally it should be mentioned that an extensive amount of standardization is applied to all kinds of different scopes and a lot of literature is available with respect to quality management (QM) and statistics.